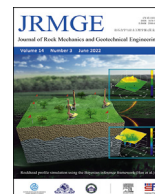




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Consolidation of partially saturated ground improved by impervious column inclusion: Governing equations and semi-analytical solutions

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ABSTRACT

This study focuses on the consolidation behavior and mathematical interpretation of partially-saturated ground improved by impervious column inclusion. The constitutive relations for soil skeleton, pore air and pore water for partially saturated soils are proposed in the context of partially-saturated ground improved by impervious column inclusion. Settlement equation and dissipation equations of excess pore air/water pressures for a partially saturated improved ground are then derived. The semi-analytical solutions for ground settlement and pore pressure dissipation are then obtained through the Laplace transform and validated by the existing solutions for two special cases in the literature and the numerical results obtained from the finite difference method. A series of parametric studies is finally conducted to investigate the influence of some key factors on consolidation of partially saturated ground improved by impervious column inclusion. Based on the parametric study, it can be found that a higher value of the area replacement ratio or modulus of the pile results in a longer dissipation time of excess pore air pressure (PAP), a shorter dissipation time of excess pore water pressure (PWP), and a lower normalized settlement.

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1. Introduction

Any soils near the ground surface that exist in a relatively dry environment can be partially saturated (Sheng, 2011). Partially saturated soil is therefore inevitably encountered in geotechnical engineering, such as ground treatment and improvement which perhaps need to be conducted before the construction of super structure. Inclusion of the granular or concrete piles in either fully saturated or partially saturated soils, which together form an improved ground, has been popularly used to enhance the strength and control the displacement of the ground. Meanwhile, the composite ground has been adopted widely to improve ground bearing capacity and accelerate the consolidation of soils (Wang and Xu, 2013). If external loads are applied, the improved ground consolidates no matter the soils in the ground are fully or partially saturated. Hence, for a better design, it is essential to provide a solid

theoretical basis for the design of unsaturated ground treatment with impervious column inclusion.

For saturated conditions, the analytical and numerical analyses on the consolidation of saturated ground improved by permeable or impervious columns have been developed by extending Terzaghi's consolidation framework (Barron, 1948; Hansbo, 1980). Due to different materials used for the columns in the improved ground, the consolidation analysis for the reinforced ground under saturated states can be divided into two categories: (1) reinforced with the permeable columns, for example, granular piles (Han and Ye, 2002; Zhang et al., 2006, 2008; Wang, 2009; Hu, 2013; Mujah et al., 2016; Lu et al., 2017a, b; Wang et al., 2019); and (2) improved by impervious columns, for example, concrete piles (Lorenzo and Bergado, 2003; Xie et al., 2009a; Wijerathna et al., 2017; Lu et al., 2018; Yang et al., 2019, 2021; Lang and Yang, 2020). Besides, Lu et al. (2020) obtained a series of analytical solutions for the consolidation of ground improved by composite columns consisting of an impervious core surrounded by a pervious shell. As reviewed above, the study on the consolidation of the improved ground by impervious or permeable column inclusion under saturated states has been well performed since Terzaghi's

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consolidation theory has been widely accepted for saturated soils and served as foundation for developing consolidation equations for the saturated ground with column inclusion. However, unsaturated soil is more general and covers saturated soil as a special case. The consolidation of unsaturated soil is more complex since it involves the flow of pore air and pore water. Compared to saturated soil, the study on the consolidation of soils partially saturated with water is much more challenging and still at its early stage. Therefore, to the best of the authors' knowledge, no existing study has addressed the consolidation problem of unsaturated ground improved by impervious column inclusion.

The study aims to establish the consolidation equations for partially saturated ground improved by impervious column and derive the corresponding semi-analytical solutions. The semi-analytical solutions will help the engineers understand how the excess pore air/water pressure dissipates and how the settlement develops with time. The parameter analysis based on semi-analytical solutions will be helpful to achieve a better design for ground improvement by including impervious columns in unsaturated natural ground.

On account of the constitutive relations with two independent variables (Fredlund and Morgenstern, 1976) for volume change in partially saturated soils and the assumption of pore air/water continuity, Fredlund and his collaborators (Fredlund and Hasan, 1979; Dakshanamurthy and Fredlund, 1980; Darkshanamurthy et al., 1984) presented a relatively complete set of consolidation theory for unsaturated soils. Although using different mathematical methods, most of the studies on partially saturated soil consolidation analysis (Qin et al., 2008; Shan et al., 2012; Ho et al., 2014; Zhou and Zhao, 2014; Tan et al., 2022) were based on the pioneering work proposed by Fredlund and Hasan (1979). However, owing to the complexity of describing the consolidation of partially saturated soils, the analytical study on the consolidation of the improved ground by column inclusion is seldom found in the literature.

Besides, the complex load allocation between unsaturated soils and columns further increases the difficulty in the analytical study on the consolidation of improved ground with column inclusion under partially saturated states. For an improved ground, the columns no matter impervious or permeable carry the external loads together with the surrounding soil that may be partially saturated. Considering different mechanical properties of columns and partially saturated soils, the loads allocated to the soils are changing during the process of consolidation since the load allocation between columns and soils and the dissipation of pore air/water pressures are coupled with each other. Therefore, the consolidation of improved ground partially saturated with water is much more complex compared with either the consolidation of the natural partially saturated ground or consolidation of improved ground saturated with water.

Nonetheless, it should be acknowledged that the ground improvement with column inclusion is a typical ground treatment in practice (Xie et al., 2009b; Lu et al., 2010; Wang et al., 2018; Xiao et al., 2020; Alkhorshid et al., 2021) and can be applied to fully and partially saturated grounds in general. Compared to saturated grounds, partially saturated grounds are more general if taking into account vaporization in nature and dewatering in many engineering cases. Thus, it is demanding to establish a consolidation model for the improved ground including columns based on solid mechanics for partially saturated soils. It should be noted that, for the column-improved ground under partially saturated states, different kinds of columns (impermeable or impervious) lead to different flow paths and further result in distinct consolidation models. When the columns are permeable, the air and water phases can

flow along the radial and vertical directions simultaneously, the consolidation analysis should be performed in an axisymmetric condition (Wang et al., 2020). However, if the columns are impervious, the pore water and pore air flow only vertically and the one-dimensional (1D) consolidation model would be developed.

In the column-improved ground, the general constitutive relations for the partially saturated surrounding soils are established through the strain and stress equilibrium condition between columns and soils. Two dissipation equations, one for excess pore air pressure (PAP) and the other for the excess pore water pressure (PWP), are proposed by combining the given constitutive relations and the assumption on continuous flows of pore air and pore water. The semi-analytical solutions of ground settlement, excess PAP and PWP of the improved ground with impervious columns are given through the Laplace transform. The validation of the semi-analytical solutions is confirmed through the reported solutions for two special cases: 1D consolidation of partially saturated soil with no column inclusion and the saturated ground improved by impervious column inclusion. The semi-analytical solutions also match the numerical solutions obtained using the finite difference method (Xie et al., 2012). A parametric study is then conducted to analyze the characteristics of consolidation for partially saturated ground improved by impervious column inclusion, which can help apply the proposed model to the practical ground treatment in partially saturated soils. In addition, the work lays fundamentals for the consolidation analysis for partially saturated ground improved by impervious column inclusion.

2. Consolidation equations

2.1. Schematic model

A schematic model of the consolidation problem is outlined in Fig. 1. The columns can be arranged in rectangular or triangular layouts (Fig. 1a and b). The diagram of a unit cell of the ground is composed of two components: (1) the soil that is partially saturated with water, and (2) the impervious column in the soil. In Fig. 1c, the thickness of the soil layer (h) is equal to the length of impervious column; r_w is the radius of the impervious column; and r_e represents the radius of the equivalent improved area, determined by the arrangement type and distance of the impervious columns.

2.2. Assumptions and equilibrium conditions

The fundamental assumptions in this paper are: (1) The surrounding soil is partially saturated; (2) For any depth, the impervious columns and the surrounding soil deform equally in the vertical direction; (3) The impervious column is elastic and its deformation depends on the allocated external load and its modulus; (4) The external load is vertical; (5) Soil is homogeneous; (6) Pore water and soil particle cannot be compressed; (7) Pore air/water flows are independent with each other and keep continuous; (8) Pore air/water only flow vertically; and (9) The compressibility and air/water permeability of soil keep unchanged.

For the column-improved ground, the load is jointly carried by the surrounding soil and the column when a changing vertical external load $\Delta\sigma(t)$ acts on the ground surface. The vertical strains in the improved ground, the column, and the surrounding soil should be identical (Han and Ye, 2001; Lorenzo and Bergado, 2003; Yang et al., 2014). Specifically, the strain and stress equilibrium condition can be expressed as

$$(1 - m)\Delta\bar{\sigma}_s + m\Delta\bar{\sigma}_p = \Delta\sigma(t) \quad (1)$$

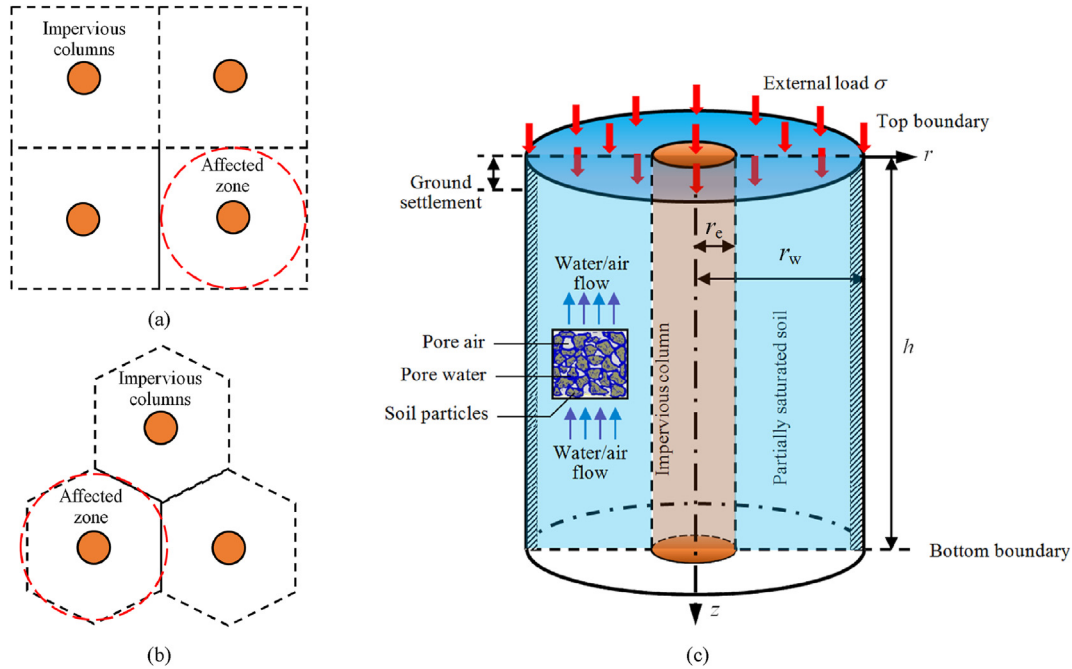


Fig. 1. Schematic diagram of partially saturated ground improved by impervious column inclusion: (a) Rectangular layout, (b) triangular layout, and (c) a unit cell of the column-improved ground.

$$\varepsilon = \varepsilon_s = \varepsilon_p \quad (2)$$

where $\Delta\bar{\sigma}_p$ and $\Delta\bar{\sigma}_s$ are the average vertical stresses within the column and partially saturated surrounding soils, respectively; $m = A_p/A_u$ is the area replacement ratio in which A_p is the column sectional area and A_u is the sectional area of the unit cell; ε_p , ε_s and ε are the vertical strains of column, partially saturated surrounding soils and unit cell, respectively, and ε_p can be determined by

$$\varepsilon_p = \frac{\Delta\bar{\sigma}_p}{E_p} \quad (3)$$

where E_p is the constrained modulus of the impervious column.

When the soil is partially saturated, the total volume change of soil consists of the volume change related to pore air and that related to pore water. By assuming soil particles and water phase are incompressible, the total volume change ratio ($\Delta V_v/V_0$), that is equal to the vertical strain (ε_s) for 1D cases, can be written as

$$\varepsilon_s = \varepsilon_a + \varepsilon_w = \frac{\Delta V_v}{V_0} = \frac{\Delta V_a}{V_0} + \frac{\Delta V_w}{V_0} \quad (4)$$

where ΔV_v , ΔV_a and ΔV_w are the changes in the volume of pore, pore air and pore water, respectively; V_0 is the initial volume of a partially saturated soil element; and ε_a and ε_w are the strains of air and water phases, respectively.

The constitutive equations for the partially saturated soil have been proposed by Fredlund and Morgenstern (1976), which are adopted in this study. These equations are under a K_0 -loading condition and can be written as

$$\frac{\Delta V_v}{V_0} = m_{1k}^s \Delta(\bar{\sigma}_s - u_a) + m_2^s \Delta(u_a - u_w) \quad (5)$$

$$\frac{\Delta V_a}{V_0} = m_{1k}^a \Delta(\bar{\sigma}_s - u_a) + m_2^a \Delta(u_a - u_w) \quad (6)$$

$$\frac{\Delta V_w}{V_0} = m_{1k}^w \Delta(\bar{\sigma}_s - u_a) + m_2^w \Delta(u_a - u_w) \quad (7)$$

where m_{1k}^s , m_{1k}^a and m_{1k}^w are the volume change coefficients for the soil, pore air and pore water, respectively, subjected to a change in soil net stress $\Delta(\bar{\sigma}_s - u_a)$, and $m_{1k}^s = m_{1k}^a + m_{1k}^w$; u_a and u_w are the excess PAP and PWP, respectively; m_2^s , m_2^a and m_2^w are the volume change coefficients for the soil, pore air and pore water, respectively, subjected to a change in suction $\Delta(u_a - u_w)$, and $m_2^s = m_2^a + m_2^w$.

2.3. Constitutive relations for pore air and pore water

Rearranging Eq. (1) gives

$$\Delta\bar{\sigma}_p = \frac{\Delta\sigma(t) - (1-m)\Delta\bar{\sigma}_s}{m} \quad (8)$$

Rewriting Eq. (5) produces

$$\bar{\sigma}_s = \frac{\varepsilon_s - m_2^s \Delta(u_a - u_w)}{m_{1k}^s} + u_a \quad (9)$$

Combining Eqs. (2) and (3) leads to

$$\Delta\bar{\sigma}_p = \varepsilon_s E_p \quad (10)$$

By submitting Eqs. (9) and (10) into Eq. (8), we have

$$\varepsilon_s = m_{1k}^s \frac{\Delta[\sigma(t) - (1-m)u_a]}{m_{1k}^s m E_p + (1-m)} + m_2^s \frac{(1-m)\Delta(u_a - u_w)}{m_{1k}^s m E_p + (1-m)} \quad (11)$$

Eq. (11) can be rewritten as

$$\varepsilon_s = \frac{m_{1k}^s}{m_0} \Delta \left[\frac{\sigma(t)}{1-m} - u_a \right] + \frac{m_2^s}{m_0} \Delta (u_a - u_w) \quad (12)$$

where $m_0 = m_{1k}^s m_{Ep}/(1-m) + 1$. Conducting the first-order partial derivative of Eq. (12) with respect to t gives

$$\frac{\partial \varepsilon_s}{\partial t} = \frac{m_{1k}^s}{m_0} \frac{\partial \left[\frac{\sigma(t)}{1-m} - u_a \right]}{\partial t} + \frac{m_2^s}{m_0} \frac{\partial (u_a - u_w)}{\partial t} \quad (13)$$

According to Eqs. (4)–(7), Eq. (13) can be separated as constitutive equations in terms of pore air and pore water as

$$\frac{\partial \varepsilon_a}{\partial t} = \frac{m_{1k}^a}{m_0} \frac{\partial \left[\frac{\sigma(t)}{1-m} - u_a \right]}{\partial t} + \frac{m_2^a}{m_0} \frac{\partial (u_a - u_w)}{\partial t} \quad (14)$$

$$\frac{\partial \varepsilon_w}{\partial t} = \frac{m_{1k}^w}{m_0} \frac{\partial \left[\frac{\sigma(t)}{1-m} - u_a \right]}{\partial t} + \frac{m_2^w}{m_0} \frac{\partial (u_a - u_w)}{\partial t} \quad (15)$$

Eqs. (13)–(15) are the constitutive equations of soil, pore air and pore water, respectively, in the column-improved ground.

2.4. Dissipation equations for pore air and pore water

Based on the continuity assumption for pore air and Fick's law that can be used for the air flow in soil pores (Fredlund and Hasan, 1979), the relationship between ε_a and u_a can be expressed as

$$\frac{\partial \varepsilon_a}{\partial t} = \frac{\partial (V_a/V_0)}{\partial t} = \frac{k_a R T}{g \bar{u}_a^0 M} \frac{\partial^2 u_a}{\partial z^2} - \frac{u_{atm} n_0 (1 - S_{r0})}{\bar{u}_a^0} \frac{\partial u_a}{\partial t} \quad (16)$$

where u_{atm} is the atmospheric pressure, g is the gravitational acceleration ($g = 9.8 \text{ m/s}^2$), k_a is the permeability coefficient of air, T is the absolute temperature, n_0 is the initial porosity, M is the molecular mass of air ($M = 0.029 \text{ kg/mol}$), R is the universal gas constant ($R = 8.314 \text{ J/(mol K)}$), S_{r0} is the initial degree of saturation, \bar{u}_a^0 is the absolute pore-air pressure ($\bar{u}_a^0 = u_a^0 + u_{atm}$), and z is the depth.

Combining Eqs. (14) and (16) gives

$$\begin{aligned} \frac{\partial u_a}{\partial t} = & \frac{m_2^a}{\left[\frac{u_{atm} n_0 (1 - S_{r0})}{\bar{u}_a^0} - \frac{m_{1k}^a - m_2^a}{m_0} \right] m_0} \frac{\partial u_w}{\partial t} \\ & + \frac{k_a R T}{\left[\frac{u_{atm} n_0 (1 - S_{r0})}{\bar{u}_a^0} - \frac{m_{1k}^a - m_2^a}{m_0} \right] g \bar{u}_a^0 M} \frac{\partial^2 u_a}{\partial z^2} \\ & - \frac{m_{1k}^a}{\left[\frac{u_{atm} n_0 (1 - S_{r0})}{\bar{u}_a^0} - \frac{m_{1k}^a - m_2^a}{m_0} \right] (1 - m) m_0} \frac{\partial \sigma(t)}{\partial t} \end{aligned} \quad (17)$$

Given the continuity assumption for pore water and Darcy's law that is suitable for water flow in soil pores (Fredlund and Hasan, 1979), the relationship between ε_w and u_w can be represented using the following equation:

$$\frac{\partial \varepsilon_w}{\partial t} = \frac{\partial (V_w/V_0)}{\partial t} = \frac{k_w}{\rho_w g} \frac{\partial^2 u_w}{\partial z^2} + \frac{1}{\rho_w g} \frac{\partial k_w}{\partial z} \frac{\partial u_w}{\partial z} + \frac{\partial k_w}{\partial z} \quad (18)$$

where ρ_w is the density of water, and k_w is the permeability coefficient of water. k_w is a function of suction (i.e. $u_a - u_w$). Combining Eqs. (15) and (18) gives

$$\begin{aligned} \frac{\partial u_w}{\partial t} = & \left(\frac{m_2^w - m_{1k}^w}{m_2^w} \right) \frac{\partial u_a}{\partial t} - \frac{m_0 k_w}{m_2^w \rho_w g} \frac{\partial^2 u_w}{\partial z^2} - \frac{m_0}{m_2^w \rho_w g} \frac{\partial k_w}{\partial z} \frac{\partial u_w}{\partial z} \\ & - \frac{m_0}{m_2^w} \frac{\partial k_w}{\partial z} + \frac{m_{1k}^w}{m_2^w} \frac{1}{1 - m} \frac{\partial \sigma(t)}{\partial t} \end{aligned} \quad (19)$$

When k_w is deemed as a constant, Eq. (19) can be simplified as

$$\frac{\partial u_w}{\partial t} = - \frac{m_{1k}^w - m_2^w}{m_2^w} \frac{\partial u_a}{\partial t} - \frac{m_0 k_w}{m_2^w \rho_w g} \frac{\partial^2 u_w}{\partial z^2} + \frac{m_{1k}^w}{(1 - m) m_2^w} \frac{\partial \sigma(t)}{\partial t} \quad (20)$$

Eqs. (17) and (20) can be further simplified as

$$\frac{\partial u_a}{\partial t} = - C_{ac} \frac{\partial u_w}{\partial t} - C_{vc}^a \frac{\partial^2 u_a}{\partial z^2} + C_{ac}^\sigma \frac{\partial \sigma(t)}{\partial t} \quad (21)$$

$$\frac{\partial u_w}{\partial t} = - C_{wc} \frac{\partial u_a}{\partial t} - C_{vc}^w \frac{\partial^2 u_w}{\partial z^2} + C_{wc}^\sigma \frac{\partial \sigma(t)}{\partial t} \quad (22)$$

where C_{ac} and C_{wc} are the interactive constants with respect to the air and water phases, respectively; C_{vc}^a and C_{ac}^σ are the consolidation coefficients for the air phase; and C_{vc}^w and C_{wc}^σ are the consolidation coefficients for the water phase. The consolidation parameters can be expressed as

$$C_{ac} = \frac{m_2^a}{m_0 \left[\frac{m_{1k}^a - m_2^a}{m_0} - \frac{u_{atm} n_0 (1 - S_{r0})}{\bar{u}_a^0} \right]} \quad (23a)$$

$$C_{vc}^a = \frac{k_a R T}{g \bar{u}_a^0 M \left[\frac{m_{1k}^a - m_2^a}{m_0} - \frac{u_{atm} n_0 (1 - S_{r0})}{\bar{u}_a^0} \right]} \quad (23b)$$

$$C_{ac}^\sigma = \frac{-m_{1k}^a}{(1 - m) m_0 \left[\frac{u_{atm} n_0 (1 - S_{r0})}{\bar{u}_a^0} - \frac{m_{1k}^a - m_2^a}{m_0} \right]} \quad (23c)$$

$$C_{wc} = \frac{m_{1k}^w - m_2^w}{m_2^w} \quad (23d)$$

$$C_{vc}^w = \frac{m_0 k_w}{m_2^w \rho_w g} \quad (23e)$$

$$C_{wc}^\sigma = \frac{m_{1k}^w}{m_2^w (1 - m)} \quad (23f)$$

Eqs. (21) and (22) are the dissipation equations of excess PAP and PWP, respectively. When the external load $\sigma(t)$ is an instantaneous load, Eqs. (21) and (22) can be simplified as

$$\frac{\partial u_a}{\partial t} = - C_{ac} \frac{\partial u_w}{\partial t} - C_{vc}^a \frac{\partial^2 u_a}{\partial z^2} \quad (24)$$

$$\frac{\partial u_w}{\partial t} = - C_{wc} \frac{\partial u_a}{\partial t} - C_{vc}^w \frac{\partial^2 u_w}{\partial z^2} \quad (25)$$

Note that the permeability coefficients should be the functions of degree of saturation and void ratio, and the moduli for the partially saturated soils and columns are also not constant in general. However, to achieve semi-analytical solutions, it is acceptable to assume that these parameters are constant for most of the analytical consolidation analyses.

3. Semi-analytical solution through the Laplace transform

3.1. Boundary and initial conditions

Boundary conditions are undoubtedly an important part of the study of consolidation problems, which affect the flow of air and water phases in soils. In this paper, the top boundary is homogeneous and permeable but the bottom boundary is impermeable. This boundary condition is widely used for both unsaturated soil consolidation and composite ground settlement. Considering the engineering practice in the ground treatment, more complex boundaries (such as partially permeable top boundary) can be considered in the subsequent studies. However, applying more complex boundary conditions may bring more difficulties to derive semi-analytical solutions.

The boundary conditions are given as

$$u_w(0, t) = 0, u_a(0, t) = 0 \text{ (top boundary)} \quad (26a)$$

$$\frac{\partial u_w(h, t)}{\partial z} = 0, \frac{\partial u_a(h, t)}{\partial z} = 0 \text{ (bottom boundary)} \quad (26b)$$

The initial conditions are

$$u_a(z, 0) = u_a^0 \quad (27a)$$

$$u_w(z, 0) = u_w^0 \quad (27b)$$

where u_a^0 and u_w^0 are the initial excess PAP and PWP, respectively.

3.2. Solutions for dissipation equations in the Laplace domain

The dissipation equations for the partially saturated ground with impervious column inclusion are in the form of second-order partial differential equations with two variables. Based on the previous study (Wang et al., 2017), the general solutions to Eqs. (24) and (25) can be given as

$$\tilde{u}_a(z, s) = -C_1 a_4 e^{\xi z} - C_2 a_4 e^{-\xi z} - D_1 a_5 e^{\eta z} - D_2 a_5 e^{-\eta z} + \frac{u_a^0}{s} \quad (28)$$

$$\tilde{u}_w(z, s) = C_1 e^{\xi z} + C_2 e^{-\xi z} + D_1 e^{\eta z} + D_2 e^{-\eta z} + \frac{u_w^0}{s} \quad (29)$$

where

$$\xi = \sqrt{(-a_2 - \sqrt{a_2^2 - 4a_1 a_3}) / (2a_1)}$$

$$\eta = \sqrt{(-a_2 + \sqrt{a_2^2 - 4a_1 a_3}) / (2a_1)}$$

$$a_1 = C_{vc}^a C_{vc}^w / (s C_{wc})$$

$$a_2 = (C_{vc}^a + C_{vc}^w) / C_{wc}$$

$$a_3 = s(1 - C_{ac} C_{wc}) / C_{wc}$$

$$a_4 = C_{vc}^w z^2 / (s C_{wc}) + 1 / C_{wc}$$

$$a_5 = C_{vc}^w \eta^2 / (s C_{wc}) + 1 / C_{wc}$$

where C_1, C_2, D_1 and D_2 are the arbitrary functions with respect to s , which are determined by the given drainage boundary conditions.

Applying the Laplace transform to the boundary conditions described by Eq. (26) and submitting Eqs. (28) and (29) into the result of the Laplace transform of Eq. (26), a set of semi-analytical solutions can be achieved as follows:

$$\tilde{u}_a(z, s) = -\frac{a_4(u_a^0 + a_5 u_w^0) \cosh[\xi(z-h)]}{s(a_4 - a_5) \cosh(\xi h)} + \frac{a_5(u_a^0 + a_4 u_w^0) \cosh[\eta(z-h)]}{s(a_4 - a_5) \cosh(\eta h)} + \frac{u_a^0}{s} \quad (30)$$

$$\tilde{u}_w(z, s) = \frac{(u_a^0 + a_5 u_w^0) \cosh[\xi(z-h)]}{s(a_4 - a_5) \cosh(\xi h)} - \frac{(u_a^0 + a_4 u_w^0) \cosh[\eta(z-h)]}{s(a_4 - a_5) \cosh(\eta h)} + \frac{u_w^0}{s} \quad (31)$$

3.3. Solutions for ground settlement in the Laplace domain

The settlement equation for the partially saturated ground with impervious column inclusion has been given in Eq. (13). Conducting the Laplace transform to Eq. (13) yields

$$\tilde{\varepsilon}_s = \frac{m_{1k}^s}{m_0(1-m)} \left[\sigma(s) - \frac{\sigma(0)}{s} \right] + \frac{m_2^s - m_{1k}^s}{m_0} \tilde{u}_a - \frac{m_2^s}{m_0} \tilde{u}_w - \frac{(m_2^s - m_{1k}^s) u_a^0}{s m_0} + \frac{m_2^s u_w^0}{s m_0} \quad (32)$$

Considering the applied load is a constant, Eq. (32) can be simplified as

$$\tilde{\varepsilon}_s = \frac{m_2^s - m_{1k}^s}{m_0} \tilde{u}_a - \frac{m_2^s}{m_0} \tilde{u}_w - \frac{(m_2^s - m_{1k}^s) u_a^0 - m_2^s u_w^0}{s m_0} \quad (33)$$

In the Laplace domain, the settlement of the partially saturated soil can be written as

$$\tilde{w}(s) = \int_0^h \tilde{\varepsilon}_s(z, s) dz \quad (34)$$

Substituting Eqs. (30) and (31) into Eq. (34) leads to the following settlement equation in the Laplace domain:

$$\tilde{w}(s) = \varphi_1 \tanh(\xi h) + \varphi_2 \tanh(\eta h) \quad (35)$$

$$\text{where } \varphi_1 = -\frac{(u_a^0 + a_5 u_w^0)[a_4(m_2^s - m_{1k}^s) + m_2^s]}{s m_0(a_4 - a_5)\xi} \text{ and } \varphi_2 = \frac{(u_a^0 + a_4 u_w^0)[a_5(m_2^s - m_{1k}^s) + m_2^s]}{s m_0(a_4 - a_5)\eta}$$

3.4. Solutions in the time domain

The inverse Laplace transform on the semi-analytical solutions about $\tilde{u}_a(z, s)$, $\tilde{u}_w(z, s)$ and $\tilde{w}(s)$ in the Laplace transform domain, i.e. Eqs. (30), (31) and (35), are conducted by the method proposed by Crump (1976). Details of Crump's method can be found in Appendix. In the time domain, the solutions are

$$u_a(z, t_j) \approx \frac{e^{\alpha t_j}}{\tau} \frac{1}{2} \tilde{u}_a(z, a) - \sum_{k=1}^{+\infty} \left\{ \operatorname{Re} \left[\tilde{u}_a \left(z, a + \frac{k\pi i}{\tau} \right) \right] \cos \frac{k\pi t_j}{\tau} \right.$$

$$-\operatorname{Im}\left[\tilde{u}_a\left(z, a+\frac{k\pi i}{\tau}\right)\right] \sin \frac{k\pi t_j}{\tau}\} \quad (36)$$

$$u_w(z, t_j) = \frac{e^{\alpha t_j}}{\tau} \frac{1}{2} \tilde{u}_w(z, a) - \sum_{k=1}^{+\infty} \left\{ \operatorname{Re}\left[\tilde{u}_a\left(z, a+\frac{k\pi i}{\tau}\right)\right] \cos \frac{k\pi t_j}{\tau} - \operatorname{Im}\left[\tilde{u}_a\left(z, a+\frac{k\pi i}{\tau}\right)\right] \sin \frac{k\pi t_j}{\tau} \right\} \quad (37)$$

$$w(t_j) \approx \frac{e^{\alpha t_j}}{\tau} \frac{1}{2} \tilde{w}(a) - \sum_{k=1}^{+\infty} \left\{ \operatorname{Re}\left[\tilde{w}\left(a+\frac{k\pi i}{\tau}\right)\right] \cos \frac{k\pi t_j}{\tau} - \operatorname{Im}\left[\tilde{w}\left(a+\frac{k\pi i}{\tau}\right)\right] \sin \frac{k\pi t_j}{\tau} \right\} \quad (38)$$

where $a = \alpha_b - \ln(0.1E_r)/(2\tau)$, α_b should be specified equal to, or slightly larger than the value of α ; E_r is the required relative error in the values of the inverse Laplace transform ($0 \leq E_r < 1$); the values of t_j ($j = 1, 2, \dots, n$) must be provided in a monotonically increasing order.

To calculate the specific results of excess PAP, excess PWP and settlements, the computation scheme is also composed to perform the inverse Laplace transform.

4. Verification

The settlement of the partially saturated ground improved by impervious column is dependent on the dissipation of pore pressures (both excess PAP and PWP). Therefore, valid dissipation equations of excess PAP and PWP are of key importance to the consolidation process of the column-improved ground. The verification for the proposed dissipation equations and the corresponding solutions is presented here. Two special cases are employed to check the proposed dissipation equations. In addition, the numerical solutions from the finite difference method are employed to check the validity of the corresponding solutions. The verification by special case can be obtained by giving a special value to a parameter based on the proposed solutions. The verification by special case provides necessary but not sufficient condition to the proposed solutions. The advantage of the finite difference method is that it can directly transform the differential problem into an algebraic one. However, the solution by finite difference method is an approximate solution in mathematics. By using two methods jointly, the correctness and soundness of the solution obtained in this paper can be well verified.

4.1. Special cases

The 1D consolidation for partially saturated soils without column inclusion and 1D consolidation of the saturated ground improved by impervious columns are two special cases for the proposed dissipation equations (i.e. Eqs. (21) and (22)). Here, we check the compatibility of Eqs. (21) and (22) against these two special cases.

The partially saturated ground would show different improvements if the different area replacement ratios (m) are adopted. Especially, when the area replacement ratio m is equal to 0, the ground is a partially saturated soil deposit without any improvement. Considering the intermediate variable m_0 is equal to 1, we have the following equations by submitting $m = 0$ and $m_0 = 1$ into Eqs. (23a)–(23f).

$$C_{a0} = \frac{m_2^a}{m_{1k}^a - m_2^a - \frac{u_{atm}n_0(1-S_{r0})}{\bar{u}_a^{\alpha^2}}} \quad (39a)$$

$$C_{v0}^a = \frac{k_a RT}{g \bar{u}_a^0 M \left[m_{1k}^a - m_2^a - \frac{u_{atm}n_0(1-S_{r0})}{\bar{u}_a^{\alpha^2}} \right]} \quad (39b)$$

$$C_{a0}^\sigma = \frac{m_{1k}^a}{\frac{u_{atm}n_0(1-S_{r0})}{\bar{u}_a^{\alpha^2}} - m_{1k}^a + m_2^a} \quad (39c)$$

$$C_{w0} = \frac{m_{1k}^w - m_2^w}{m_2^w} \quad (39d)$$

$$C_{v0}^w = \frac{k_w}{m_2^w \rho_w g} \quad (39e)$$

$$C_{w0}^\sigma = \frac{m_{1k}^w}{m_2^w} \quad (39f)$$

It is clear that the dissipation coefficients in Eqs. (39a)–(39f) are the same as those in the 1D consolidation theory of partially saturated soils proposed by Fredlund and Hasan (1979). Therefore, it can be concluded that the dissipation equations for the partially saturated ground improved by impervious column proposed here can degenerate into the equations for partially saturated soils that have been widely accepted in the literature.

When the surrounding soil is saturated, we have $m_{1k}^a = m_2^a = 0$, $m_{1k}^w = m_2^w = 1/E_s$ and $u_a = 0$. Herein, E_s is the compressive modulus of saturated soil under the K_0 -loading condition. Eq. (24) is unnecessary, and Eq. (25) can be changed to

$$\frac{\partial u_w}{\partial t} = - \frac{[mE_p + (1-m)E_s]k_w}{(1-m)\rho_w g} \frac{\partial^2 u_w}{\partial z^2} \quad (40)$$

Correspondingly, the equations in the present paper can be reduced to the 1D consolidation (dissipation) equation of improved ground in saturated soils. Eqs. (1)–(3) and (8) and (10) are the same for the cases of partially saturated and fully saturated soils. For saturated soils, Eq. (4) should be rewritten as

$$\varepsilon_s = \frac{\Delta \bar{\sigma}_s - u_w}{E_s} \quad (41)$$

Rearranging Eq. (41) leads to

$$\Delta \bar{\sigma}_s = E_s \varepsilon_s + u_w \quad (42)$$

Submitting Eqs. (10) and (41) into Eq. (8) gives

$$\varepsilon_v = \frac{\Delta[\sigma(t) - (1-m)u_w]}{mE_p + (1-m)E_s} \quad (43)$$

Here, it is also assumed that the external load $\sigma(t)$ is a constant. Then, applying the first-order partial derivative of Eq. (43) with respect to t gives

$$\frac{\partial \varepsilon_v}{\partial t} = - \frac{1-m}{mE_p + (1-m)E_s} \frac{\partial u_w}{\partial t} \quad (44)$$

Based on the continuity assumption and Darcy's law for water phase in saturated soils, the following equation can be produced:

Table 1

The parameters for consolidation analysis.

Parameter	Value	Parameter	Value
H	10 m	n_0	50%
S_{r0}	80%	k_w	10^{-10} m/s
E_p	-1×10^{-5} kPa	m	0.05
m_{1k}^s	-2.5×10^{-5} kPa $^{-1}$	m_{1k}^w	-0.5×10^{-5} kPa $^{-1}$
m_2^s	-1×10^{-5} kPa $^{-1}$	m_2^w	-2×10^{-5} kPa $^{-1}$
q_0	100 kPa	u_a^0	20 kPa
u_w^0	40 kPa	u_{atm}	101.3 kPa

$$\frac{\partial \varepsilon_v}{\partial t} = \frac{k_w}{\rho_w g} \frac{\partial^2 u_w}{\partial z^2} \quad (45)$$

Combining Eqs. (44) and (45) gives

$$\frac{\partial u_w}{\partial t} = - \frac{k_w [mE_p + (1-m)E_s]}{(1-m)\rho_w g} \frac{\partial^2 u_w}{\partial z^2} \quad (46)$$

It is clear that Eqs. (40) and (46) are identical to each other. Therefore, it can be concluded that the partially saturated ground improved by impervious column proposed here can strictly degenerate into the equations for the saturated column-improved ground.

The semi-analytical solutions for the above two special cases in the literature are employed to validate the solutions obtained in this study. Here, we define the normalized settlement as the ratio of the settlement of the improved partially saturated ground with impervious columns to that of natural partially saturated ground (Qin et al., 2008). For model verification, the parameters are listed in Table 1.

In addition, it should be noted that since the values of moduli like m_{1k}^s , m_2^s , m_{1k}^w and m_2^w are negative, the value of E_p is negative correspondingly. Fig. 2 shows the comparison between the semi-analytical solutions (see solid curves) in terms of excess PAP and PWP (u_a/u_a^0 and u_w/u_w^0) obtained in the study and the solutions (see dots) for 1D consolidation of partially saturated soils in the literature (Qin et al., 2008). Fig. 3 shows the comparison between the semi-analytical solutions proposed in this paper and the existing solutions (Yang et al., 2014) for the saturated ground including impervious columns. All the solutions obtained here match with the benchmarks in the literature well.

4.2. Numerical solution by finite difference method

The dissipation equations of partially saturated ground improved by impervious columns can also be solved using the finite difference method. Herein an explicit forward difference technique is used, and the finite difference mesh is illustrated in Fig. 4.

Considering the similar work reported by Fredlund and Hassan (1979) when solving the 1D consolidation equations of partially saturated soils, the consolidation equations in this paper (see Eqs. (24) and (25)) are directly given in the form of the finite difference as below:

$$u_a(i, j+1) = u_a(i, j) + \frac{\beta_a f_1^a}{1 - C_{ac} C_{wc}} - \frac{C_{ac}}{1 - C_{ac} C_{wc}} \beta_w g_1^w \quad (47)$$

$$u_w(i, j+1) = u_w(i, j) + \frac{\beta_w g_1^w}{1 - C_{ac} C_{wc}} - \frac{C_{wc}}{1 - C_{ac} C_{wc}} \beta_a f_1^a \quad (48)$$

where $\beta_w = C_{wc}^w \Delta t / \Delta z^2$, $\beta_a = C_{ac}^a \Delta t / \Delta z^2$, $g_1^w = u_w(i+1, j) - 2u_w(i, j) + u_w(i-1, j)$, and $f_1^a = u_a(i+1, j) - 2u_a(i, j) + u_a(i-1, j)$.

The initial condition is given as

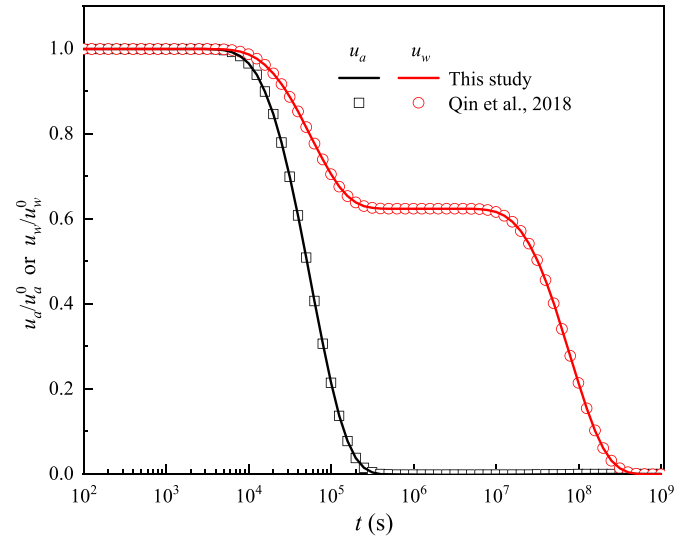


Fig. 2. Semi-analytical solutions of relative excess PAP/PWP for the special case in this paper and solutions from the literature (Qin et al., 2008).

$$u_w(i, 1) = u_w^0, u_a(i, 1) = u_a^0 \quad (i = 1, 2, 3, \dots, z_n + 1) \quad (49)$$

The top boundary conditions are as follows:

$$u_a(i, 1) = 0 \quad (50)$$

$$u_w(i, 1) = 0 \quad (51)$$

The bottom boundary conditions are given below:

$$u_a(i, z_n + 1) = u_a(i, z_n + 1) - \frac{\beta_{ax}}{1 - C_{ac} C_{wc}} f_t^a + \frac{C_{ac}}{1 - C_{ac} C_{wc}} \beta_{wx} f_t^w - \frac{\beta_{az}}{1 - C_{ac} C_{wc}} g_t^a + \frac{C_{ac}}{1 - C_{ac} C_{wc}} \beta_{wz} g_t^w \quad (52)$$

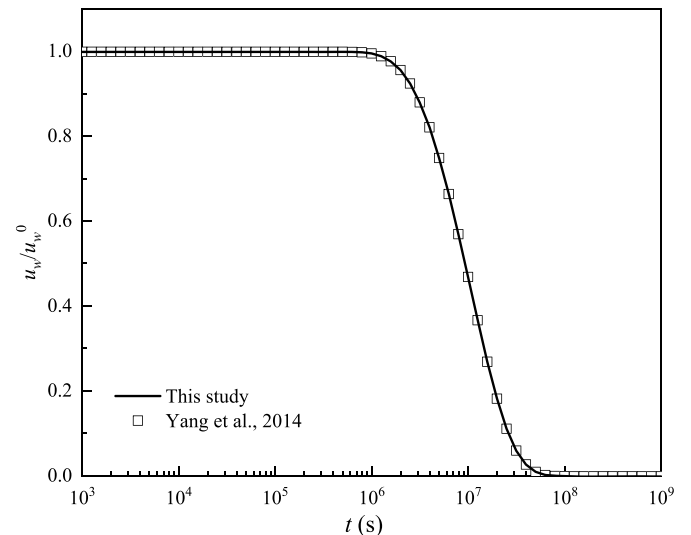


Fig. 3. Semi-analytical solutions of relative excess PWP for the special case in this paper and solutions from the literature (Yang et al., 2014).

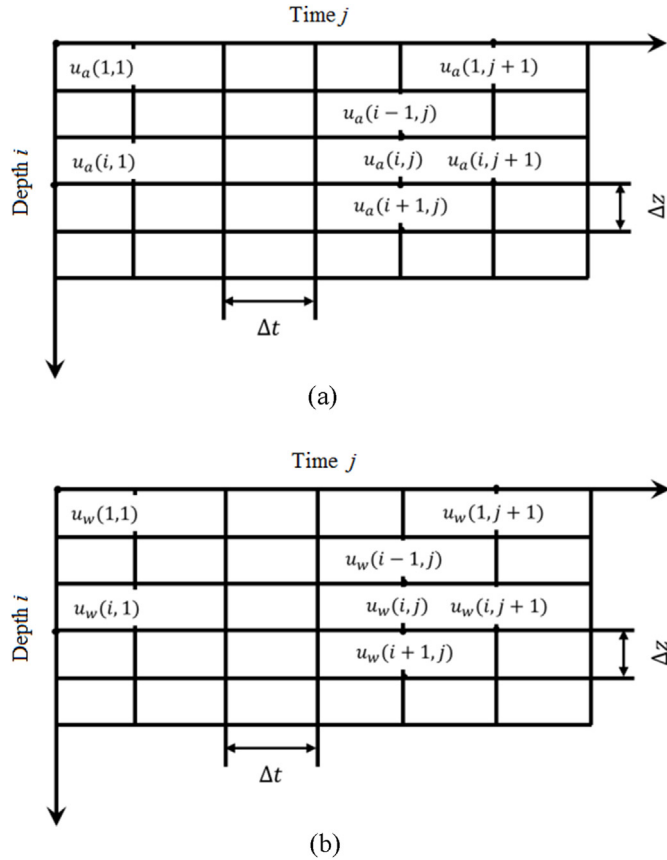


Fig. 4. Finite difference mesh for solving the dissipation equations: (a) PAP and (b) PWP.

$$u_w(i, z_n + 1) = u_w(i, z_n + 1) - \frac{\beta_{wx}}{1 - C_{ac}C_{wc}} f_t^w + \frac{C_{wc}}{1 - C_{ac}C_{wc}} \beta_{ax} f_t^a - \frac{\beta_{wz}}{1 - C_{ac}C_{wc}} g_t^w + \frac{C_{wc}}{1 - C_{ac}C_{wc}} \beta_{az} g_t^a \quad (53)$$

where z_n is the total number of the grid along the direction of depth, z_{n+1} is the bottom, $\beta_{ax} = C_{vc}^a \Delta t / \Delta x^2$, $\beta_{az} = C_{vc}^a \Delta t /$

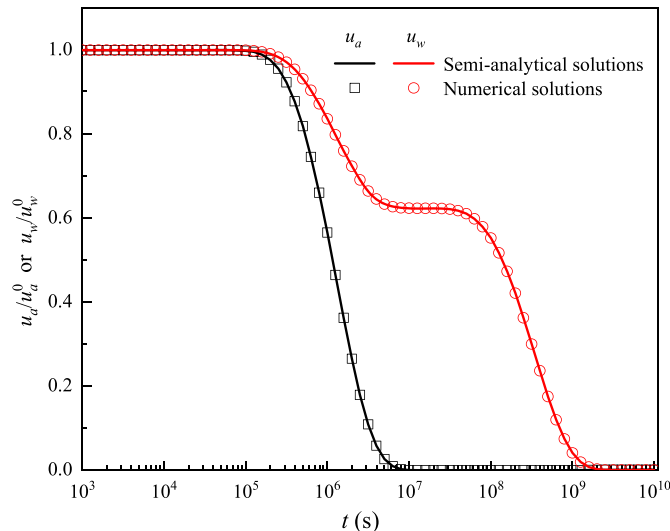


Fig. 5. Comparisons of relative excess PAP/PWP of semi-analytical and numerical solutions for partially saturated ground improved by impervious column inclusion.

Δz^2 , $\beta_{wx} = C_{vc}^w \Delta t / \Delta x^2$, $\beta_{wz} = C_{vc}^w \Delta t / \Delta z^2$, $C_{vc}^a \Delta t / \Delta x^2$, $f_t^a = u_a(i+1, z_n + 1) - 2u_a(i, z_n + 1) + u_a(i-1, z_n + 1)$, $f_t^w = u_w(i+1, z_n + 1) - 2u_w(i, z_n + 1) + u_w(i-1, z_n + 1)$, $g_t^a = -u_a(i, z_n + 1) + u_a(i, z_n)$, and $g_t^w = -u_w(i, z_n + 1) + u_w(i, z_n)$. By using Eqs. (47)–(53), a computational program can be compiled to obtain the numerical solutions to dissipation equations for partially saturated ground improved with impervious columns under an instantaneous load. The semi-analytical solutions are also compared to the numerical ones obtained by the finite difference method (Fig. 5). The semi-analytical solutions obtained in this study agree well with the numerical solutions.

Fig. 6a and b shows the excess PAP change and the excess PWP change at various depths ($0 \leq z/h \leq 1$) and time ($10^3 \text{ s} \leq t \leq 10^9 \text{ s}$), respectively. The parameters of surrounding soil are listed in Table 1. When far away from the permeable boundary, the excess PAP and PWP need more time to be dissipated completely. The dissipation of excess PAP is faster than that of excess PWP. When $z/h = 1$, the completion time for air flow is less than $2 \times 10^6 \text{ s}$, but that for water flow is more than 108 s.

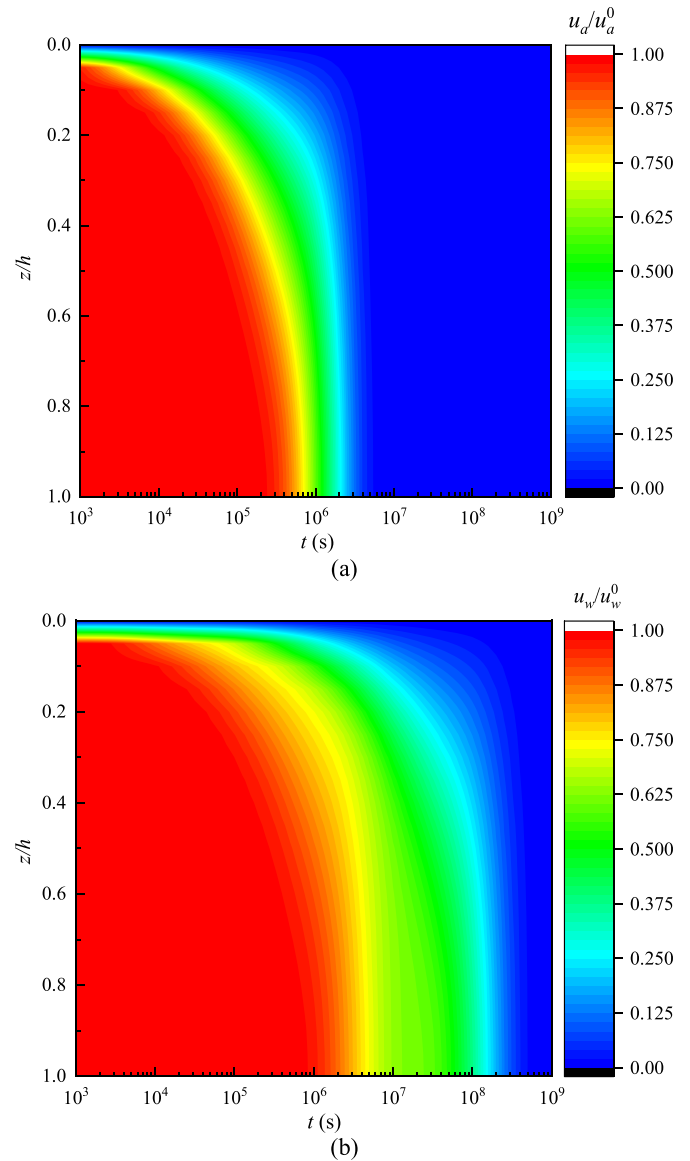


Fig. 6. Excess pore pressure dissipation along the depth: (a) Excess PAP and (b) excess PWP.

5. Parametric studies

Parametric studies are performed to clarify the influences of the parameter s related to the impervious column and partially saturated soil on u_a/u_a^0 (the relative excess PAP), u_w/u_w^0 (the relative excess PWP) and w^* (normalized settlement). The parameters used here are listed in Table 1.

5.1. Influence of different parameters of the impervious column on ground consolidation

The influence of two key parameters of the column on the consolidation of partially saturated ground improved by impervious column is investigated in this section. For the first scenario, the area replacement ratio (m) ranges from 0% to 20%, while $E_p = 1 \times 10^5$ kPa, $k_w = 10^{-10}$ m/s and $k_a = 10^{-9}$ m/s. For the second scenario, $m = 5\%$, $k_w = 10^{-10}$ m/s and $k_a = 10^{-9}$ m/s, while the constrained modulus of the pile (E_p) ranges from 1×10^5 kPa to 5×10^5 kPa.

5.1.1. Area replacement ratio

The area replacement ratio (m) reflects the extent of improvement of the ground by impervious columns, the value of which depends on the diameter, distance and arrangement of columns. When the external load is given, different area replacement ratios represent different stress allocations of external load to surrounding soils and columns for the column-improved ground. For instance, when a smaller value of m is adopted (i.e. the column diameter is small or the distance between the columns is large), the surrounding soils will carry more external load. Fig. 7 shows the variation in relative excess PAP/PWP and normalized settlement with various values of area replacement ratios (m), respectively. As described in Fig. 7a, a greater value of m results in a longer dissipation time of the excess PAP. As shown in Fig. 7b, considering the cross point of u_w/u_w^0 curves, the whole dissipation curves can be separated into two stages. At the first stage, dissipation of relative excess PWP occurs simultaneously with the dissipation of relative excess PAP, and u_w/u_w^0 curves show a similar feature compared with the u_a/u_a^0 curves (i.e. the dissipations of relative excess PAP and PWP become slower with a higher value of area replacement ratio). For some cases with a lower area replacement ratio (e.g. $m = 0\%$ and 5%), the dissipation of relative excess PAP has been completed at the first stage and u_w/u_w^0 curves show a plateau at the beginning of the second stage. For the cases with a higher area replacement ratio (e.g. $m = 15\%$ and 20%), more loads are carried by impervious columns. The fewer loads acting on the surrounding soil make the dissipation of relative excess PAP more difficult. The dissipation of relative excess PAP lasts at the second stage, which occurs simultaneously with the dissipation of relative excess PAP as same as at the first stage. For these cases, no distinct plateau of u_w/u_w^0 curves can be observed. It is clear that applying a higher external load to a partially saturated soil layer will induce a larger settlement. The normalized settlement at $m = 5\%$ reduces to about 40% of that at $m = 0$ (Fig. 7c). It is clear that when $m = 0$, this case is equivalent to the 1D consolidation of partially saturated soil.

5.1.2. Constrained modulus of the impervious column (E_p)

Fig. 8 shows the variations in relative excess PAP/PWP and normalized settlement of the improved grounds by impervious columns with various values of E_p . By comparing the results induced by different area replacement ratios (m) and column's constrained moduli (E_p), it can be found that there are similar variations in the relative excess PAP/PWP and normalized settlement. As a constant area replacement ratio is adopted (e.g. $m = 5\%$),

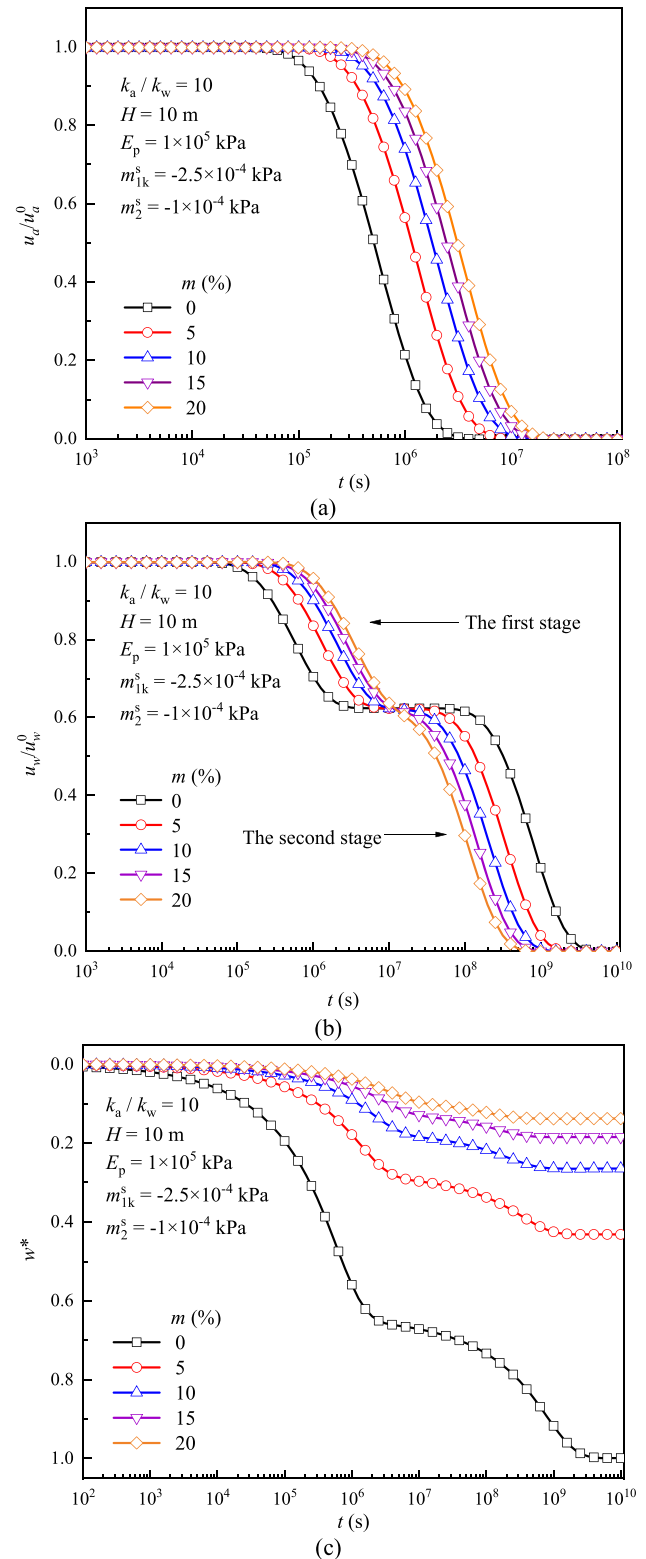
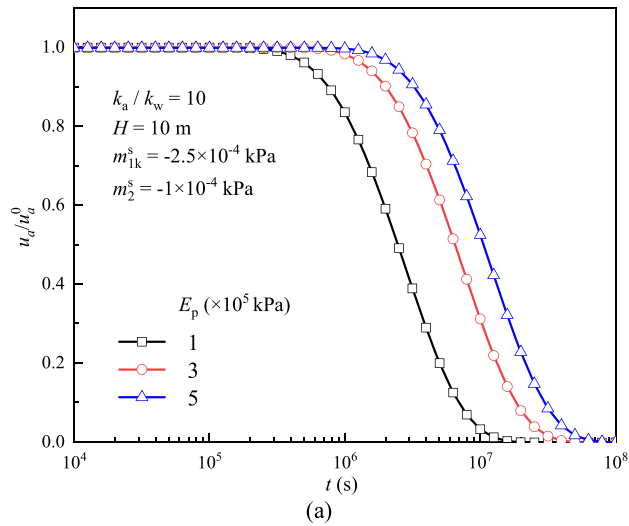
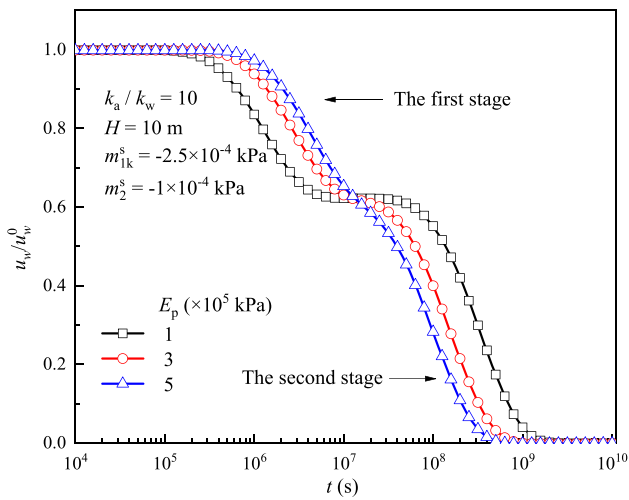


Fig. 7. Consolidation of a partially saturated ground with impervious column inclusion with different area replacement ratios (m): (a) Relative excess PAP, (b) relative excess PWP, and (c) normalized settlement.

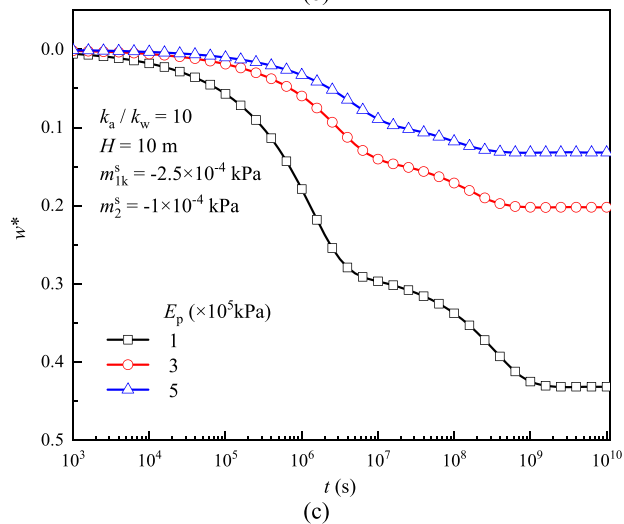
a greater value of E_p (i.e. stronger pile) leads to a slower dissipation of relative excess PAP (Fig. 8a) but a less normalized settlement



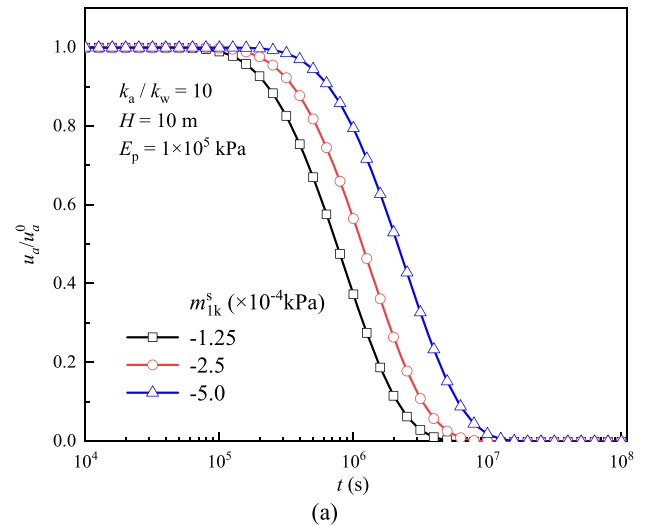
(a)



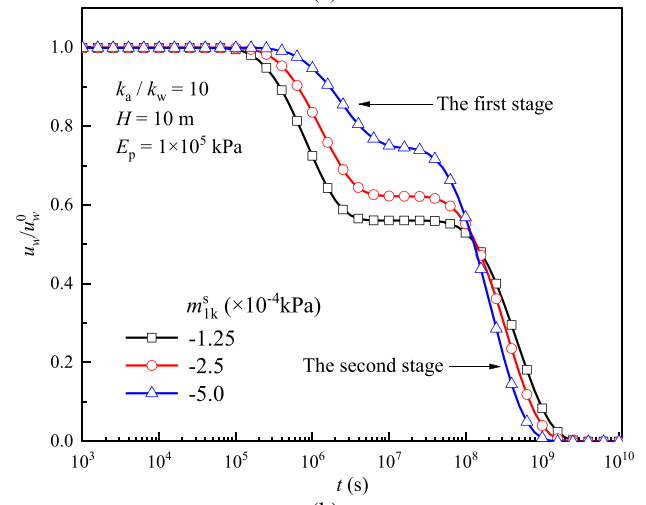
(b)



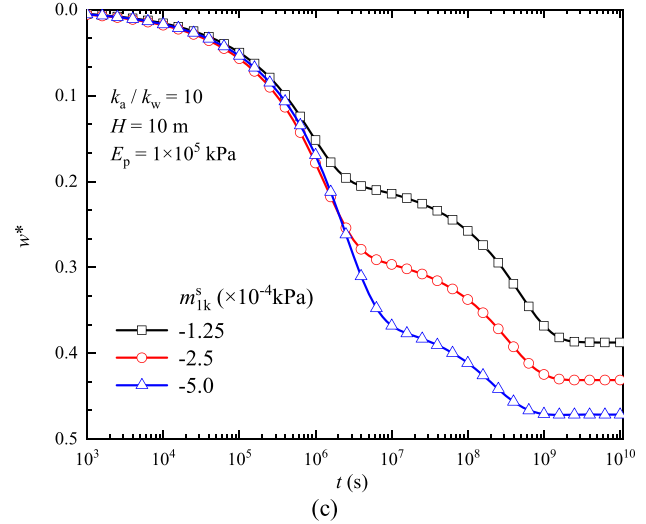
(c)



(a)



(b)



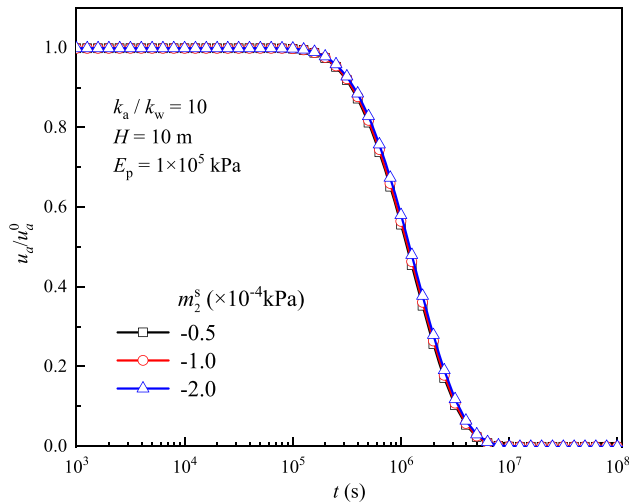
(c)

Fig. 8. Consolidation of a partially saturated ground with impervious column inclusion with different constrained moduli of the pile (E_p): (a) Relative excess PAP, (b) relative excess PWP, and (c) normalized settlement.

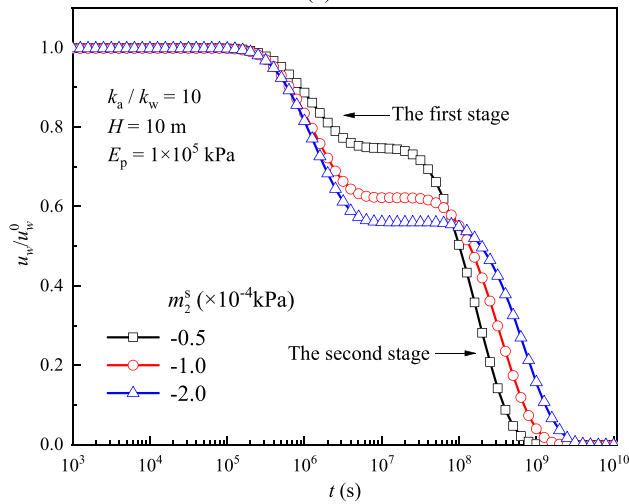
Fig. 9. Consolidation of a partially saturated ground by impervious column inclusion with different coefficients of volume change (m_{1k}^s): (a) Relative excess PAP, (b) relative excess PWP, and (c) normalized settlement.

(Fig. 8c). Similar to Fig. 7b, we can observe two distinct stages for the dissipation of relative excess PWP (Fig. 8b). The plateau of u_w/u_w^0

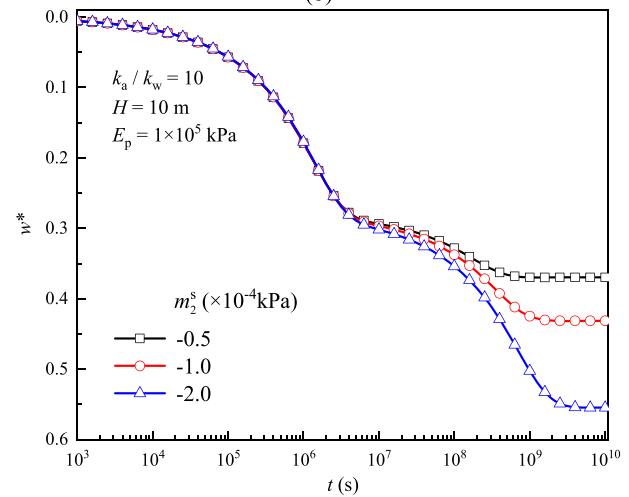
curves become less distinct when columns become stronger (i.e. increase of E_p).



(a)

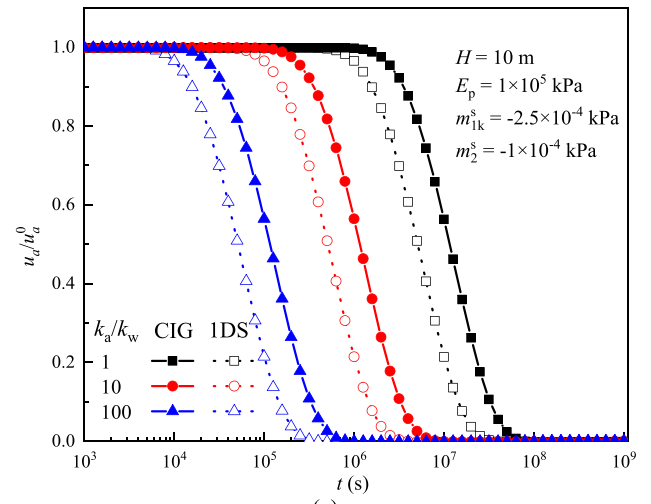


(b)

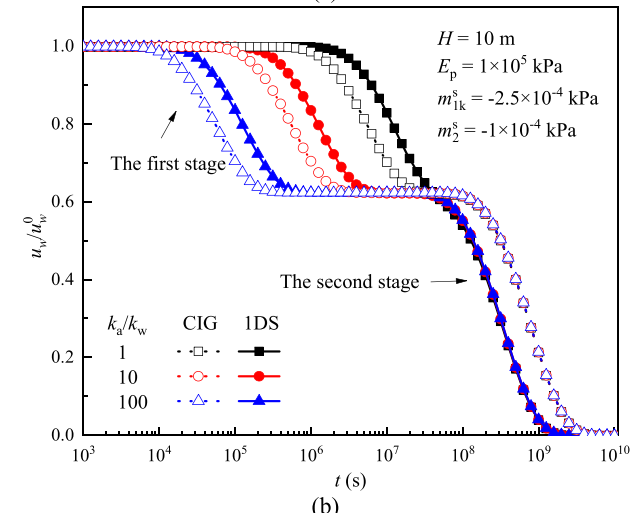


(c)

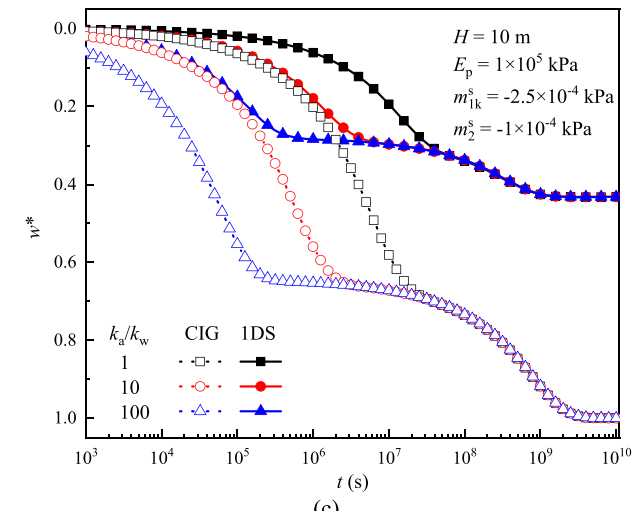
Fig. 10. Consolidation of partially saturated ground improved by impervious column with different coefficients of volume change (m_2^s): (a) Relative excess PAP, (b) relative excess PWP, and (c) normalized settlement.



(a)



(b)



(c)

Fig. 11. Consolidation of partially saturated ground improved by impervious column with different ratios of permeability coefficient (k_a/k_w): (a) Relative excess PAP, (b) relative excess PWP, and (c) normalized settlement.

5.2. Influence of different parameters of the partially saturated soil on ground consolidation

The consolidation properties of the partially saturated ground improved by impervious column are discussed when adopting different parameters of the partially saturated soil, such as m_{1k}^s , m_2^s and k_a/k_w . When analyzing the effects of m_{1k}^s and m_2^s , we set up $E_p = 1 \times 10^5$ kPa, $k_a = 10k_w = 10^{-9}$ m/s, and $m = 5\%$. When studying the effect of k_a/k_w , we keep k_w as a constant ($= 10^{-10}$ m/s), but change k_a to achieve different ratios of k_a/k_w from 1 to 100.

5.2.1. Volume change coefficient for the soil element subjected to net stress change (m_{1k}^s)

Fig. 9 illustrates the variations of the relative excess PAP/PWP and the normalized settlement with different coefficients m_{1k}^s . As shown in Fig. 9a, increasing the absolute value of m_{1k}^s will delay the dissipation of the relative excess PAP. Meanwhile, before the completion of dissipation of the relative excess PAP, similar dissipation characteristics can be observed for both relative excess PAP and PWP. But when the dissipation of relative excess PWP enters the second stage, the dissipation of relative excess PWP completes more quickly as the absolute value of the m_{1k}^s becomes larger (Fig. 9b). As the result of the dissipation of relative excess PAP and PWP, increasing the absolute value of m_{1k}^s leads to a higher normalized settlement. At last, there are residual stages for the normalized settlement curves, i.e. the normalized settlement becomes unchanged when both u_a/u_a^0 and u_w/u_w^0 are completed (Fig. 9c). If a larger absolute value of m_{1k}^s is adopted, the gradient between the net normal stress ($\bar{\sigma}_s - u_a$) and the volumetric change of soils ε_s is higher. Higher compressibility of investigated soil leads to a higher normalized settlement.

5.2.2. Volume change coefficient for the soil element subjected to suction change (m_2^s)

Fig. 10 describe the changes in relative excess PAP/PWP and the normalized settlement at different volume change coefficients (m_2^s). There is almost no influence on the dissipation of excess PAP for different values of m_2^s (Fig. 10a). Compared Fig. 10a with Fig. 9a, we can find that, concerning the PAP dissipation, the stiffness in terms of net stress has a predominant influence compared to that in terms of suction. This result is similar to the outcomes in the literature (Wang et al., 2020) for consolidation behavior of unsaturated ground improved by permeable columns. As shown in Fig. 10b, the PWP dissipation can also be divided into two stages with a distinct plateau in the middle. Amplifying the absolute value of the volume change coefficient (m_2^s) causes more dissipation time for relative excess PWP and a larger normalized settlement (Fig. 10b and c). That is similar to the results induced by the change of m_{1k}^s , a larger absolute value of m_2^s indicates a higher gradient between the suction ($u_a - u_w$) and the volumetric change of soils ε_s , i.e. a better compressibility of investigated soil due to suction. The normalized settlement almost develops along the same path with the different values of (m_2^s) during the dissipation of relative excess PAP ($t < 107$ s), and that is consistent with effect of m_2^s on the dissipation of relative excess PAP.

5.2.3. Permeability coefficient ratio (k_a/k_w)

Fig. 11 shows the changes in the relative excess PAP/PWP and normalized settlement at different ratios of permeability coefficient (k_a/k_w). In this case, the comparison of the consolidation of the column-improved partially saturated ground (CIG) and 1D consolidation for partially saturated soils (1DS) was simultaneously carried out. With a given value of k_a/k_w , the dissipation of relative excess PAP of 1D consolidation is quicker than that of

column-improved ground (Fig. 11a). Meanwhile, there is a shorter plateau for the dissipation of relative excess PWP in the case of improved ground. In addition, the normalized settlement of the column-improved ground is almost 40% of that of 1D consolidation (Fig. 11c). That is because the external load is borne by the surrounding soils and the columns jointly for the improved ground, and there is only part of the external load applied on the investigated partially saturated surrounding soil. When different ratios of permeability coefficient (k_a/k_w) are adopted, there is a similar property for 1D consolidation and the consolidation of the column-improved ground, i.e. a greater ratio of k_a/k_w will accelerate the consolidation rate. Different dissipation processes can be found for relative excess PAP (Fig. 11a). The relative excess PWP dissipates along different paths before the plateau (Fig. 11b) and the dissipation on relative excess PWP seems to be independent upon k_a/k_w .

6. Conclusions

In this paper, the consolidation equations (including one settlement equation and two dissipation equations) for partially saturated ground improved by impervious column are proposed, based on the assumption of pore air/water flow continuity, Fick's law for pore air, and Darcy's law for pore water. Taking the advantage of Laplace transform, the semi-analytical solutions of the proposed consolidation equations are presented. Two special cases and numerical solutions were employed to verify the proposed equations and corresponding solutions. A series of parametric studies was provided to investigate the consolidation characteristics of partially saturated ground improved by impervious column inclusion. The main conclusions are drawn as follows:

- (1) The parameters for the impervious column have a great effect on the consolidation behavior of improved partially saturated ground. A higher value of either the area replacement ratio (m) or modulus of the pile (E_p) causes a longer dissipation time of excess PAP, a shorter dissipation time of excess PWP, and a lower normalized settlement. The normalized settlement at $m = 5\%$ reduces to about 40% of that at $m = 0$.
- (2) The consolidation property of partially saturated ground improved by impervious column is influenced by soil parameters distinctively. Higher values of m_{1k}^s and m_2^s lead to a larger normalized settlement. It is of interest to note that there is a significant improvement effect on the partially saturated soil ground by including impervious columns. For various soil conditions, the final settlement of column-improved ground is significantly reduced, compared to the cases without column inclusion.
- (3) The dissipation of relative excess pore-air pressure of 1D consolidation is quicker than that of column-improved ground, while the relative excess pore-water pressure dissipates along different paths before the plateau, and the normalized settlement of the column-improved ground is almost 40% of that of 1D consolidation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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List of symbols

A_p	Column sectional area
A_u	Sectional area of the unit cell of the column-improved ground
C_{ac}	Interactive constant with respect to the air phase
C_{wc}	Interactive constant with respect to the water phase
C_{vc}^a	C_c^a Consolidation coefficients for the air phase
C_{vc}^w	C_c^w Consolidation coefficients for the water phase
E_p	Constrained modulus of the impervious column
g	Gravitational acceleration
h	Thickness of soil layer
k_a	Permeability coefficient of air
k_w	Permeability coefficient of water
M	Molecular mass of air
m	Area replacement ratio
m_{1k}^a	Volume change coefficients for pore air subjected to a change in net stress ($\bar{\sigma}_s - u_a$)
m_{1k}^s	Volume change coefficients for the soil subjected to a change in net stress ($\bar{\sigma}_s - u_a$)
m_{1k}^w	Volume change coefficients for pore water subjected to a change in net stress ($\bar{\sigma}_s - u_a$)
m_2^a	Volume change coefficients for pore air subjected to a change in suction ($u_a - u_w$)
m_2^s	Volume change coefficients for the soil subjected to a change in suction ($u_a - u_w$)
m_2^w	Volume change coefficients for pore water subjected to a change in suction ($u_a - u_w$)
n_0	Initial porosity
q_0	Initial vertical load
R	Universal gas constant
r_e	Radius of the equivalent improved area
r_w	Radius of the impervious column
S_{r0}	Initial degree of saturation
T	Absolute temperature
u_a	Excess pore air pressure
u_{atm}	Atmospheric pressure
u_a^0	Initial excess pore air pressure
u_w^0	Initial excess pore water pressure
\bar{u}_a^0	Absolute pore-air pressure
u_w	Excess pore water pressure
V_v	Pore volume
V_a	Volume of pore air
V_w	Volume of pore water
V_0	Initial volume of a partially saturated soil element
w	Settlement
w^*	Normalized settlement
Z_n	Total number of the grid along the direction of depth
ρ_w	Density of water
ε_s	Vertical strain of partially saturated soil around the column
ε_p	Vertical strain of the column
ε	Vertical strain of the unit cell of the improved ground
$\sigma(t)$	Vertical external load
$\bar{\sigma}_p$	Vertical stresses within the column

$\bar{\sigma}_s$ Vertical stresses within the partially saturated surrounding soils

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jrmge.2021.09.017>.

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