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Theoretical investigation of interaction between a rectangular plate and fractional viscoelastic foundation

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ABSTRACT

The interaction between plates and foundations is a typical problem encountered in geotechnical engineering. The long-term plate performance is highly dependent on the rheological characteristics of ground soil. Compared with conventional linear rheology, the fractional calculus-based theory is a more powerful mathematical tool that can address this issue. This paper proposes a fractional Merchant model (FMM) to investigate the time-dependent behavior of a simply supported rectangular plate on viscoelastic foundation. The correspondence principle involving Laplace transforms was employed to derive the closed-form solutions of plate response under uniformly distributed load. The plate deflection, bending moment, and foundation reaction calculated using the FMM were compared with the results obtained from the analogous elastic model (EM) and the standard Merchant model (SMM). It is shown that the upper and lower bound solutions of the FMM can be determined using the EM. In addition, a parametric study was performed to examine the influences of the model parameters on the time-dependent behavior of the plate–foundation interaction problem. The results indicate that a small fractional differential order corresponds to a plate resting on a sandy soil foundation, while the fractional differential order value should be increased for a clayey soil foundation. The long-term performance of a foundation plate can be accurately simulated by varying the values of the fractional differential order and the viscosity coefficient. The observations from this study reveal that the proposed fractional model has the capability to capture the variation of plate deflection over many decades of time.

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1. Introduction

The interaction between a loaded plate and the soil foundation is a typical problem in foundation and pavement engineering. To solve the plate–foundation interaction problem, the well-known Winkler's foundation model is widely adopted (e.g. Matsunaga, 2000; Buczkowski and Torbacki, 2001; Huang and Thambiratnam, 2002; Zhong and Zhang, 2006). However, significant time-dependent phenomena of plates under surface loading have been

observed in field, which were mainly induced by the rheological properties of ground soil. In the past few decades, the behavior of a plate resting on the viscoelastic foundation has been theoretically examined by numerous studies (e.g. Nassar, 1981; Zaman et al., 1991; Sun, 2003). In the 1950s and 1960s, the Maxwell model, the Kelvin–Voigt model, and the Merchant model are three commonly used rheological models. These simple viscoelastic models have only two or three parameters and therefore the prediction accuracy is fairly poor. More model parameters were needed to make the predictions more accurate, but difficulties in determining the parameter values arose (Chen et al., 2006).

Gemant (1936) for the first time introduced the fractional constitutive models of viscoelastic materials. In the constitutive equations of the proposed models, the integer-order differential operators were replaced by fractional-order ones. Over the past few decades, the fractional derivative viscoelastic models have shown their powerfulness in describing viscoelastic behavior of materials (Welch et al., 1999; Mainardi, 2012). Up to now, there have been a very limited number of studies that used the fractional calculus-based models to solve geotechnical problems (e.g. Atanackovic and Stankovic, 2004; Dikmen, 2005), especially the plate–

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foundation interaction problem. The main reason for this problem is the complexity involved in numerical analysis of fractional models. In the studies of Yin et al. (2007, 2013), a single fractional derivative element was proposed to describe the rheological properties of soils and rocks under different loading conditions. Zhu et al. (2011, 2012) established a fractional model by replacing the dashpot in the standard Kelvin–Voigt model with the fractional element. This model was used to analyze the ground deformation and the plate performance. One apparent drawback of this model is that it cannot account for the instantaneous deflection for a loaded plate on the viscoelastic foundation. Therefore, a more advanced fractional model with a reasonable number of parameters is necessary.

In this paper, a fractional Merchant model (FMM) is proposed to describe the time-dependent plate–foundation interaction problem. The solutions of plate deflection, bending moment, and foundation reaction are presented and compared with the calculated results of elastic and standard viscoelastic models. Through the analyses of a numerical example, the effectiveness of this four-parameter model is verified. A parametric study is then undertaken to examine the influences of the model parameters on the predicted results.

2. Fractional Merchant model (FMM)

2.1. Basics of fractional calculus

The n th derivative of a function $f(t)$ is expressed as $D^n f(t) = d^n f(t)/dt^n$. If n is replaced by a fraction, this expression becomes a fractional derivative. Fractional calculus is usually expressed in terms of Riemann–Liouville definition. The Riemann–Liouville fractional integration of function $f(t)$ of order ν (Miller and Ross, 1993) is defined as

$${}_0 D_t^{-\nu} f(t) = \frac{1}{\Gamma(\nu)} \int_0^t (t - \xi)^{\nu-1} f(\xi) d\xi \quad (\text{Re}(\nu) > 0, t > 0) \quad (1)$$

where the subscripts 0 and t at the left and right sides of D refer to the limits of the integration; $\Gamma(\nu)$ is the Gamma function with argument ν . Let $[\alpha]$ be the smallest integer that exceeds α , the Riemann–Liouville fractional derivative of order α (Miller and Ross, 1993) is

$${}_0 D_t^\alpha f(t) = {}_0 D_t^{[\alpha]} [{}_0 D_t^{-\nu} f(t)] \quad (\text{Re}(\alpha) > 0, t > 0) \quad (2)$$

where $\nu = [\alpha] - \alpha > 0$. In the following derivation, the fractional derivative of the Riemann–Liouville type of order α is denoted as D_{RL}^α .

2.2. Generalization of the FMM

In the theoretical rheology, the relationships between stress $\sigma(t)$ and strain $\epsilon(t)$ of a spring and a dashpot can be expressed in terms of differential operators:

$$\left. \begin{aligned} \sigma(t) &= E D_{RL}^0 \epsilon(t) \\ \epsilon(t) &= \eta D_{RL}^1 \epsilon(t) \end{aligned} \right\} \quad (3)$$

where E and η are the elastic modulus and viscosity coefficient, respectively.

The fractional rheological models are on the basis of an element called “intermediate model” by Smit and de Vries (1970), or “spring-pot” by Koeller (1984). The fractional derivative element shown in

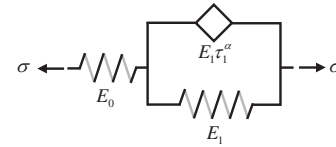


Fig. 1. Four-parameter FMM.

Fig. 1 is represented by a diamond, which has been adopted by many scholars (Bagley and Torvik, 1979; Welch et al., 1999; Dikmen, 2005). Let $\tau = \eta/E$ be the creep time, the constitutive equation of the fractional derivative element can be expressed as

$$\sigma(t) = E \tau^\alpha D_{RL}^\alpha \epsilon(t) \quad (0 \leq \alpha \leq 1) \quad (4)$$

where D^α is the fractional differentiation defined by Eq. (2). It is noted that for $\alpha = 0$, the model defined by Eq. (4) is a spring. In the case of $\alpha = 1$, Eq. (4) can be the constitutive equation of a dashpot. The coefficient α is therefore considered to be a dimensionless parameter concerning the memory of materials (Koeller, 1984).

The Merchant model consists of a Kelvin–Voigt model and a spring connected in series. As shown in Fig. 1, if the dashpot in the Kelvin–Voigt model is replaced by a fractional derivative element, the FMM is obtained. The stress–strain relationship of this model can be expressed as

$$E_0 (D_{RL}^\alpha + 1/\tau_1^\alpha) \epsilon(t) = (D_{RL}^\alpha + 1/t_1^\alpha) \sigma(t) \quad (5)$$

where $\tau_1 = \eta/E_1$, $t_1 = \tau_1 / \sqrt[4]{1 + E_0/E_1}$. It is obvious that if $\alpha = 1$, the FMM collapses to the standard Merchant model (SMM).

3. Closed-form solutions using the fractional soil–foundation interaction model

As shown in Fig. 2, a rectangular plate rests on a fractional Merchant foundation with an average thickness of d . The plate is simply supported on all four edges and is subjected to a uniformly distributed load of q_0 . The length, width and thickness of this plate are a , b and h , respectively. The governing equation for plate deflection $w(x, y)$ is

$$D \nabla^2 \nabla^2 w(x, y) + R(x, y) = q_0 \quad (6)$$

where $R(x, y)$ is the foundation reaction; D is the flexural rigidity of the plate defined by

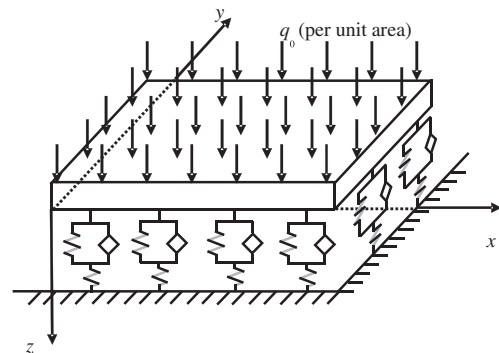


Fig. 2. Schematic illustration of a loaded rectangular plate resting on a fractional Merchant foundation.

$$D = \frac{Eh^3}{12(1-\mu^2)} \quad (7)$$

where μ is the Poisson's ratio of the plate.

The solutions of bending moments M_x and M_y of the foundation plate are

$$\left. \begin{aligned} M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \\ M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) \end{aligned} \right\} \quad (8)$$

3.1. Elastic solution

For a plate resting on a Winkler-type foundation consisting of elastic springs with stiffness $k = E/d$, the reaction of the foundation is

$$R(x, y) = kw(x, y) \quad (9)$$

The plate deflection can be derived from Eq. (6) (Timoshenko and Woinowsky-Krieger, 1959) as

$$w(x, y) = \frac{16q_0}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{(2m-1)\pi x}{a} \sin \frac{(2n-1)\pi y}{b}}{(2m-1)(2n-1) \left\{ \pi^4 D \left[\left(\frac{2m-1}{a} \right)^2 + \left(\frac{2n-1}{b} \right)^2 \right]^2 + k \right\}} \quad (10)$$

where $m, n = 1, 2, 3, \dots$

Taking the Laplace transforms of Eqs. (6), (9) and (10), we obtain the governing equation and the resulting deflection expressed in the “s” domain:

$$D\nabla^2 \nabla^2 \bar{w}(x, y, s) + k\bar{w}(x, y, s) = \frac{q_0}{s} \quad (11)$$

$$\bar{w}(x, y, s) = \frac{16q_0}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{(2m-1)\pi x}{a} \sin \frac{(2n-1)\pi y}{b}}{(2m-1)(2n-1) \left\{ \pi^4 D \left[\left(\frac{2m-1}{a} \right)^2 + \left(\frac{2n-1}{b} \right)^2 \right]^2 + k \right\} s} \quad (12)$$

3.2. Viscoelastic solution

Assuming the surface load is applied on the foundation plate quasi-statically, i.e.

$$q(t) = q_0 H(t) \quad (13)$$

where $H(t)$ is the Heaviside step function.

The plate deflection $w(x, y, t)$ is given by

$$D\nabla^2 \nabla^2 w(x, y, t) + R(x, y, t) = q(t) \quad (14)$$

Here the foundation reaction $R(x, y, t)$ is governed by the FMM which satisfies:

$$k_0 (D_{RL}^\alpha + 1/\tau_1^\alpha) w(x, y, t) = (D_{RL}^\alpha + 1/t_1^\alpha) R(x, y, t) \quad (15)$$

where $\tau_1 = \eta^*/k_1$; $t_1 = \tau_1/\sqrt[3]{1+k_0/k_1}$; k_0, k_1 and η^* are the three parameters of the viscoelastic foundation defined by the FMM.

The Laplace transforms of Eqs. (13)–(15) yield:

$$D\nabla^2 \nabla^2 \bar{w}(x, y, s) + \bar{k}(s) \bar{w}(x, y, s) = \frac{q_0}{s} \quad (16)$$

$$\bar{k}(s) = k_0 \frac{s^\alpha + 1/\tau_1^\alpha}{s^\alpha + 1/t_1^\alpha} \quad (17)$$

According to the correspondence principle (Christensen, 1982), the Laplace elastic and viscoelastic equations are equivalent if the geometry and the boundary conditions are the same. The viscoelastic problem will therefore be treated in terms of the analogous elastic problem in the following derivation. Subsequently, the viscoelastic solution can be obtained by replacing k with $\bar{k}(s)$ in Eq. (12), i.e.

$$\bar{w}(x, y, s) = \frac{16q_0}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{(2m-1)\pi x}{a} \sin \frac{(2n-1)\pi y}{b}}{(2m-1)(2n-1) \left\{ \pi^4 D \left[\left(\frac{2m-1}{a} \right)^2 + \left(\frac{2n-1}{b} \right)^2 \right]^2 + k_0 \frac{s^\alpha + 1/\tau_1^\alpha}{s^\alpha + 1/t_1^\alpha} \right\} s} \quad (18)$$

Taking the inverse Laplace transform of Eq. (18), we get

$$w(x, y, t) = \frac{16q_0}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{(2m-1)\pi x}{a} \sin \frac{(2n-1)\pi y}{b}}{(2m-1)(2n-1)(f+k_0)} \left[\frac{T}{t_1^\alpha} + \left(1 - \frac{T}{t_1^\alpha}\right) E_\alpha \left(-\frac{t^\alpha}{T}\right) \right] \quad (19)$$

where $T = (f+k_0)/(f/t_1^\alpha + k_0/\tau_1^\alpha)$, $f = \pi^4 D \{ [(2m-1)/a]^2 + [(2n-1)/b]^2 \}^2$, and E_α is the Mittag-Leffler function defined as

$$E_\alpha(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\alpha n + 1)} \quad (20)$$

Similarly, the foundation reaction and the bending moments of the plate are

$$R(x, y, t) = \frac{16q_0 k_0}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{(2m-1)\pi x}{a} \sin \frac{(2n-1)\pi y}{b}}{(2m-1)(2n-1)(f+k_0)} \left[\frac{T}{t_1^\alpha} + \left(1 - \frac{T}{t_1^\alpha}\right) E_\alpha \left(-\frac{t^\alpha}{T}\right) \right] \quad (21)$$

$$\begin{aligned} M_x &= D \left[\left(\frac{2m-1}{a} \pi \right)^2 + \mu \left(\frac{2n-1}{b} \pi \right)^2 \right] w(x, y, t) \\ M_y &= D \left[\mu \left(\frac{2m-1}{a} \pi \right)^2 + \left(\frac{2n-1}{b} \pi \right)^2 \right] w(x, y, t) \end{aligned} \quad (22)$$

Eqs. (19), (21) and (22) are the solutions of deflection, foundation reaction, and bending moments of a rectangular plate resting on a fractional Merchant foundation under a uniformly distributed load. Obviously, when the fractional differential order $\alpha = 1$, the Mittag-Leffler function reduces to e^t . As a result, Eqs. (19), (21) and (22) turn into the viscoelastic solutions derived from the SMM.

4. Numerical example and analysis

4.1. Properties of the FMM in comparison with standard models

Based on a simple algorithm for evaluating the Mittag-Leffler function (Koeller, 1984), a numerical example is presented to analyze the time-dependent properties of a rectangular plate resting on a viscoelastic foundation subjected to a uniformly distributed load of 100 kPa using the FMM. Tables 1 and 2 give the values of related parameters used in this analysis. The calculated results, i.e. the distributions of plate deflection, bending moment and foundation reaction in the longitudinal direction, are presented in Figs. 3–5. It is noted that the plate deflection, bending moment and foundation reaction are symmetrical to the axis of the plate.

Fig. 3 shows the calculated results of plate deflection using the FMM on $t = 0$ d, 150 d and 1000 d, in comparison with the predictions from the EM and the SMM. All the models show that the maximum deflection always occurs at the plate center. When $t = 0$ d, the calculated deflections using the SMM and FMM correspond to the solution of the EM given in Eq. (10) where the modulus k is replaced by k_0 . In comparison with the results of the

FMM, those calculated from the SMM are smaller when $t = 150$ d but develop quickly and eventually tend to be stable ($t = 1000$ d). With elapsed time, the deflections calculated from the SMM and FMM approach the solution of the EM as well where k in Eq. (10) is replaced by $k^* = k_0 k_1 / (k_0 + k_1)$. Using the current parameters, it approximately takes 1500 d for the deflections to be stable using the SMM, while it will be much longer using the FMM.

Similar phenomena are observed for the bending moments of the plate and the foundation reactions plotted in Figs. 4 and 5. It is again demonstrated that the FMM is capable of describing the long-term performance of a plate resting on a viscoelastic foundation.

4.2. Parametric study of the FMM

As shown in Eq. (19), the proposed FMM has four parameters, i.e. the spring stiffness k_0 and k_1 , the viscosity coefficient η^* , and the fractional differential order α . When $t = 0$ d and $t \rightarrow \infty$, the plate deflections and foundation reactions can be calculated from the EM as long as the modulus k is respectively replaced by k_0 and k^* . Therefore, it is clear that k_0 and k_1 are relevant to the upper and lower bound solutions of the FMM.

The influences of the other two parameters, namely viscosity coefficient η^* and the fractional differential order α , on the deflection–time relationships are presented in Figs. 6 and 7, respectively. It is observed from Fig. 6 that the effect of η^* of the FMM on the plate deflection is similar to that of the SMM. The parameter η^* has an effect on the rate of deflecting but does not affect the initial and the ultimate deflection. With the increase of η^* , it takes more time to obtain the ultimate deflection.

Fig. 7 depicts the impact of the fractional differential order α on the time-dependent plate deflection. The fractional differential order α varies from 0 to 1 here. When $\alpha = 0$, the resulting plate deflection is permanent immediately after the load is applied and its value equals the EM solution. It seems that the development of time-dependent deflection can be divided into two stages by a characteristic point at the deflection of around 24.8 mm. In the first stage, the deflection decreases with the increase of α ; while in the second stage, the deflection increases with increasing α .

As mentioned previously, the fractional derivative element is intermediate between purely solid and purely liquid when α varies from 0 to 1. Therefore, a low value of α corresponds to a plate resting on a sandy soil foundation with a large permeability coefficient. In contrast, the permeability coefficient is smaller for a clayey soil foundation corresponding to a higher α value. By introducing the fractional differential order α , the fractional soil–foundation interaction model can simulate various cases. In particular, the fractional derivative-based Merchant model may account for the deflection of plate over many decades of time. Similar phenomena are obtained for the curves of bending moments or foundation reactions versus time.

Table 1
Properties of the rectangular plate.

Length a (m)	Width b (m)	Height h (m)	Bending rigidity D (MPa m ³)
10	10	0.4	75

Table 2
Properties of the fractional viscoelastic foundation.

Stiffness k_0 (MPa m ⁻¹)	Stiffness k_1 (MPa m ⁻¹)	Viscosity coefficient η^* (MPa d m ⁻¹)	Fractional differential order α
5	5	2500	0.7

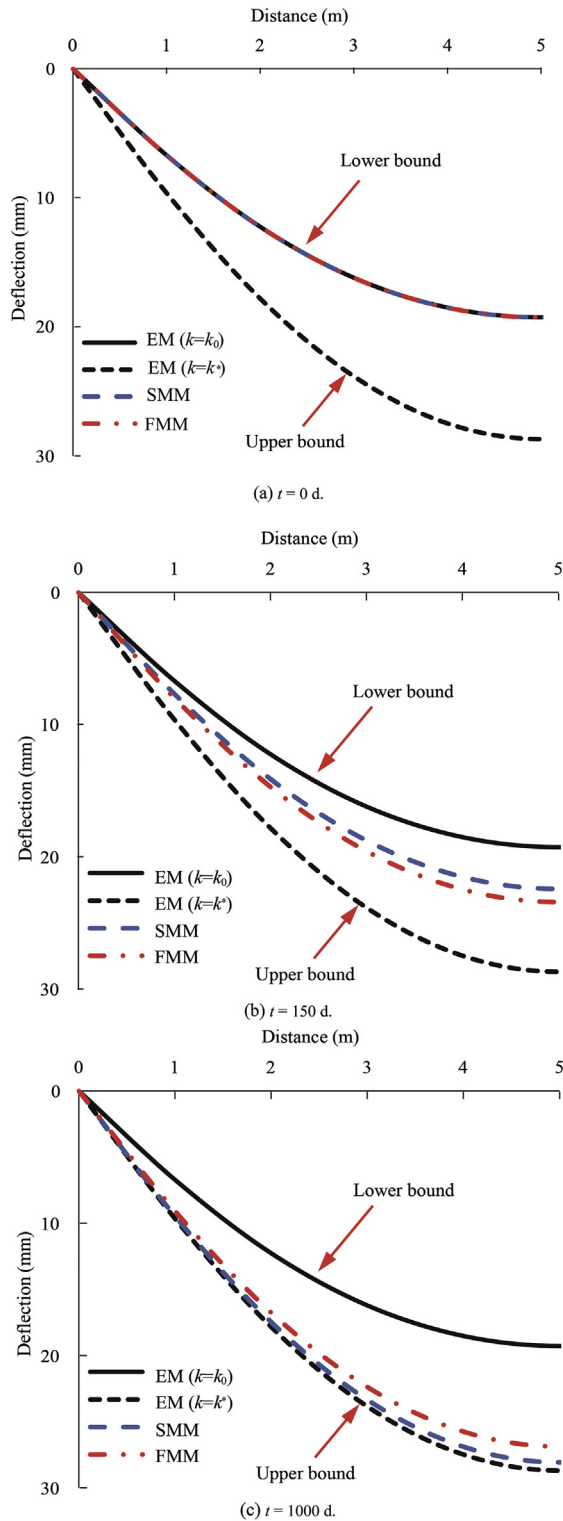


Fig. 3. Comparison of plate deflections calculated using EM, SMM and FMM.

5. Conclusions

In this study, an FMM was proposed to account for the time-dependent performance of a rectangular plate resting on a visco-elastic foundation. Closed-form solutions of plate deflection, bending moment and foundation reaction were derived using the

correspondence principle and the Laplace transform. The results calculated from the FMM were compared with those predicted using the EM and the SMM. It is found that the upper and lower bound solutions of the plate deflection, bending moment, and foundation reaction of the FMM can be obtained from the EM. The parametric study shows that the FMM can provide a wide range of

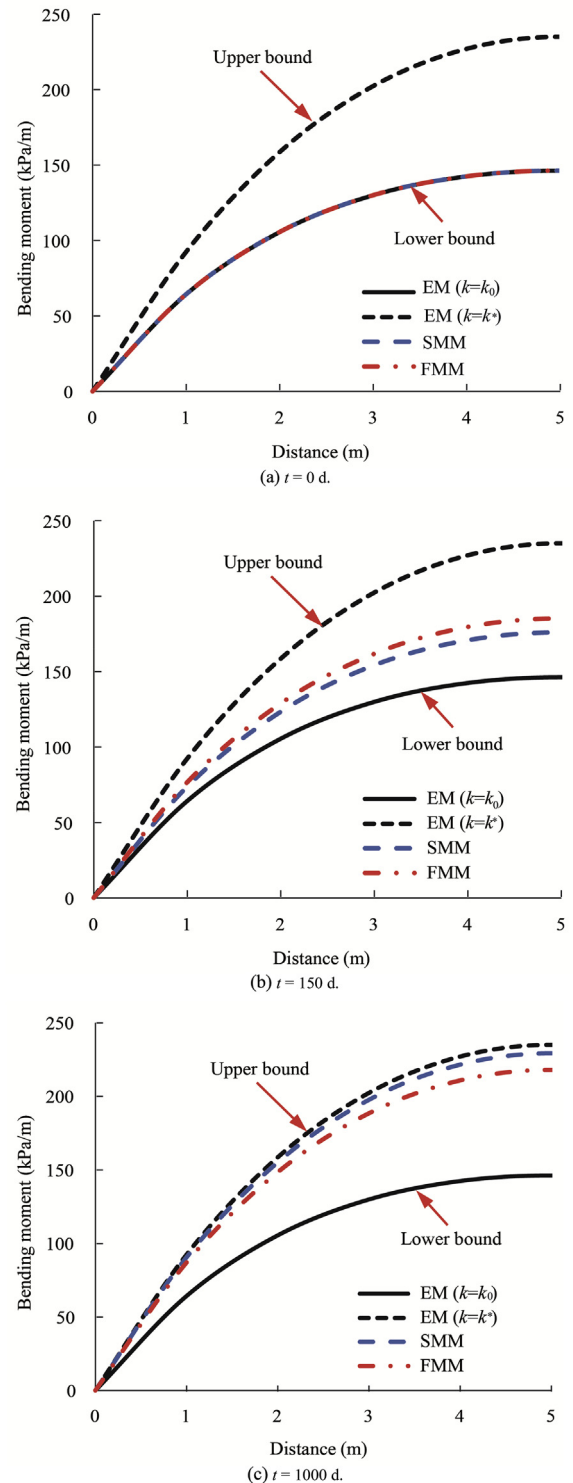


Fig. 4. Comparison of bending moments of the foundation plate calculated using EM, SMM and FMM.

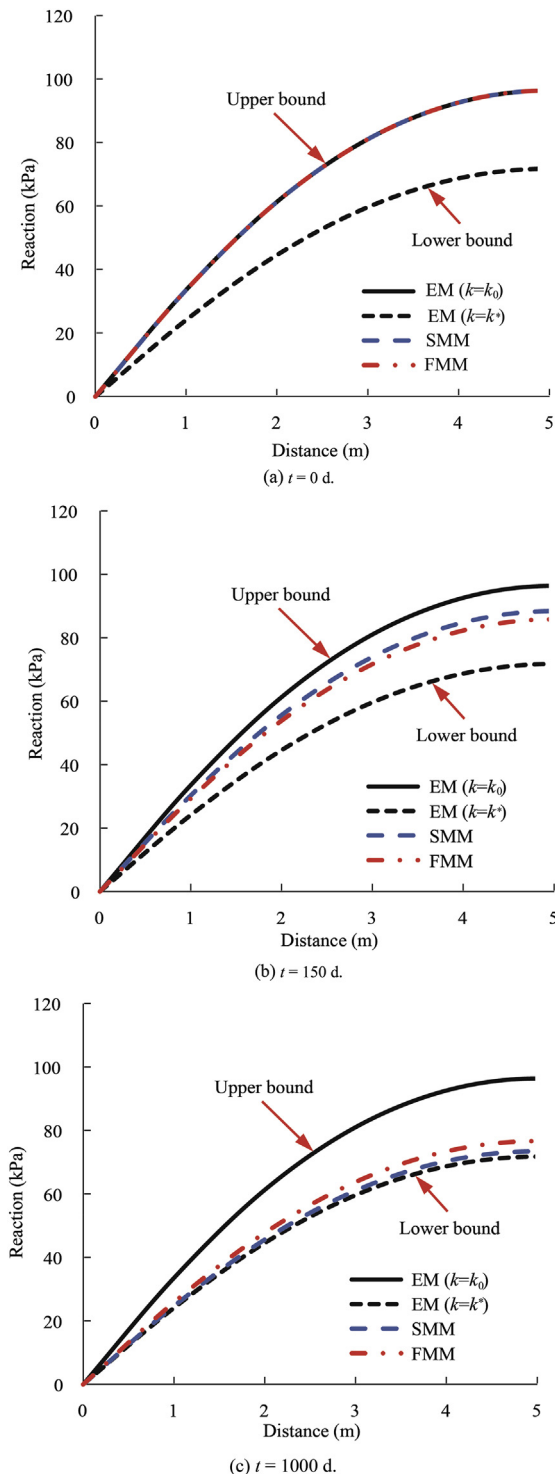


Fig. 5. Comparison of foundation reactions calculated using EM, SMM and FMM.

predictions. A small fractional differential order α corresponds to a plate resting on a sandy soil foundation characterized by a large initial deflection and a smooth deflection in the later period, while for a clayey soil foundation, the fractional differential order value should be increased. With the viscosity coefficient η^* , the introduction of the fractional differential order α provides a powerful method for describing the long-term performance of a foundation plate with a fairly small number of parameters.

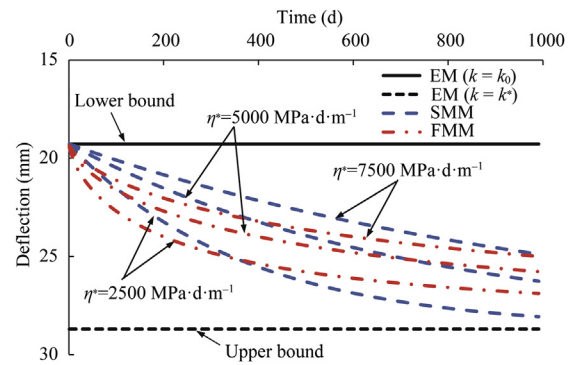


Fig. 6. Influence of viscosity coefficient η^* on the deflection–time curves at the plate center.

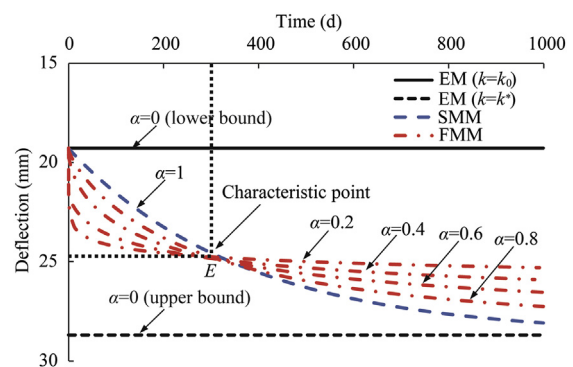


Fig. 7. Influence of fractional differential order α on the deflection–time curves at the plate center.

However, the proposed FMM was simply compared against the SMM. Further verification of the proposed model requires the support of abundant measurement data from laboratory and field experiments. Besides, the actual behavior of foundation soil is rather complicated. The plastic deformation of soil was not taken into account in the current study. More refined fractional calculus-based models should be established to investigate the foundation–plate interaction.

Conflict of interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

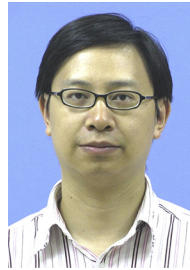
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