Appendix A. The conditional PDFs in the Gibbs sampler

In the Bayesian network, a random variable is independent with other variables given its Markov blanket consisting of its child nodes, parent nodes and other parent nodes of its child nodes (e.g. Pearl, 1988). For *μ*m, its Markov blanket is composed of $σ\_{m}^{2}$, ***t*a** and ***t*c**, leading to the simplification of the conditional PDF of *μ*m as

$f\left(μ\_{m} | σ\_{m}^{2}, B,t\_{c},Σ\_{o},T=D,t\_{a}=τ\_{a}\right)=f\left(μ\_{m} | σ\_{m}^{2},t\_{c},t\_{a}=τ\_{a}\right)$ (A1)

Eq. (A1) has a conjugate form where *μ*m remains conditional normal with the conjugate prior distribution adopted in the Bayesian network. According to Eq. (4), it can be inferred that the distribution of *t*a*j* - *t*c*j* conditional on *μ*m and $σ\_{m}^{2}$ is a normal distribution as:

$f\left(t\_{aj}-t\_{cj} | μ\_{m},σ\_{m}^{2}\right)=N\left(μ\_{m},σ\_{m}^{2}\right)$ (A2)

where N(*μ*, *σ*2) denotes the normal distribution whose mean and variance are *μ* and *σ*2, respectively. When $σ\_{m}^{2}$, ***t*a** and ***t*c** are given, *μ*m can be seen as the mean of *t*a*j* - *t*c*j*, which is normally distributed with a known variance of $σ\_{m}^{2}$. Then the conditional PDF of *μ*m can be derived through the property of the conjugate prior distribution as follows (e.g. Murphy, 2007):

$f\left(μ\_{m} | σ\_{m}^{2},B,t\_{c},Σ\_{o},T=D,t\_{a}=τ\_{a}\right)=N\left(\left(\frac{1}{σ\_{μ}^{2}}+\frac{r}{σ\_{m}^{2}}\right)^{-1}\left(\frac{μ\_{μ}}{σ\_{μ}^{2}}+\frac{1}{σ\_{m}^{2}}\sum\_{j=1}^{r}\left(τ\_{aj}-t\_{cj}\right)\right),\left(\frac{1}{σ\_{μ}^{2}}+\frac{r}{σ\_{m}^{2}}\right)^{-1}\right)$ (A3)

Similarly, it can be derived that the other conditional PDFs required in the Gibbs sampler are as follows:

$f\left(σ\_{m}^{2} | μ\_{m},B,t\_{c},Σ\_{o},T=D,t\_{a}=τ\_{a}\right)=Scale-Inv-χ^{2}\left(ν\_{m}+r, \frac{1}{ν\_{m}+r}\left(ν\_{m}s\_{m}^{2}+\sum\_{j=1}^{r}\left(τ\_{aj}-t\_{cj}-μ\_{m}\right)^{2}\right)\right)$ (A4)

$f\left(B\_{j} | μ\_{m},σ\_{m}^{2},B\_{\*j},t\_{c},Σ\_{o},T=D,t\_{a}=τ\_{a}\right)=N\left(\left(\frac{1}{σ\_{Bj}^{2}}+\sum\_{i=1}^{n\_{j}}\frac{R\_{ji}^{2}}{σ\_{oj}^{2}}\right)^{-1}\left(\frac{μ\_{Bj}}{σ\_{Bj}^{2}}+\sum\_{i=1}^{n\_{j}}\frac{R\_{ji}\left(t\_{cj}-d\_{ji}\right)}{σ\_{oj}^{2}}\right),\left(\frac{1}{σ\_{Bj}^{2}}+\sum\_{i=1}^{n\_{j}}\frac{R\_{ji}^{2}}{σ\_{oj}^{2}}\right)^{-1}\right)$ (A5)

$f\left(t\_{cj} | μ\_{m},σ\_{m}^{2},B,t\_{c\*j},Σ\_{o},T=D,t\_{a}=τ\_{a}\right)=N\left(\left(\frac{1}{σ\_{tcj}^{2}}+\frac{n\_{j}}{σ\_{oj}^{2}}+\frac{1}{σ\_{m}^{2}}\right)^{-1}\left(\frac{μ\_{tcj}}{σ\_{tcj}^{2}}+\frac{1}{σ\_{oj}^{2}}\sum\_{i=1}^{n\_{j}}\left(d\_{ji}+B\_{j}R\_{ji}\right)+\frac{τ\_{aj}-μ\_{m}}{σ\_{m}^{2}}\right),\left(\frac{1}{σ\_{tcj}^{2}}+\frac{n\_{j}}{σ\_{oj}^{2}}+\frac{1}{σ\_{m}^{2}}\right)^{-1}\right)$ (A6)

$f\left(σ\_{oj}^{2} | μ\_{m},B,t\_{c},Σ\_{o},T=D,t\_{a}=τ\_{a}\right)=Scale-Inv-χ^{2}\left(ν\_{oj}+n\_{j},\frac{1}{ν\_{oj}+n\_{j}}\left(ν\_{oj}s\_{oj}^{2}+\sum\_{i=1}^{n\_{j}}\left(d\_{ji}+B\_{j}R\_{ji}-t\_{cj}\right)^{2}\right)\right)$ (A7)

**Reference**

Murphy, K.P., 2007. Conjugate Bayesian analysis of the Gaussian distribution. Computer Science, the University of British Columbia. https://www.cs.ubc.ca/~murphyk/Papers/bayesGauss.pdf. (Accessed 25 April 2019).