**Appendix A**

Based on the associated flow rule, the strain increment tensor, deviatoric strain increment tensor, and plastic volumetric strain increment can be expressed as

,, (A1)

The consistency equation corresponding to the yield function can be expressed as

 (A2)

where

 (A3)

According to elastoplastic mechanics, the stress increment tensor can be written as

 (A4)

where *Eijkl* is the elastic stiffness tensor. The elastic modulus *E* is determined by Poisson's ratio *ν*, swelling index *κ*, initial void ratio *e*0, and current mean stress *p*.

The plastic scalar factor/ plastic multiplier ** can be obtained based on Eqs. (A2) and (A4):

 (A5)

Then, the stress increment tensor can be obtained as

 (A6)

The loading criteria are given as following in Eq. (A7), and it plays an important role in the calculation procedure. When **, i.e. the elastic unloading stage, d*R* and d*R*\* related to plastic deformation will not need to be calculated, and the elastic stiffness tensor *E*ijkl is used to calculate the stress increment. When **or **, i.e. the loading or neutral loading stage, then d*R* and d*R*\* related to plastic deformation need to be calculated, and the elastoplastic stiffness tensor  is used to calculate the stress increment.

 (A7)

**Appendix B**

The concepts of superloading surface ([Asaoka et al., 2000](#_ENREF_2)) and subloading surface ([Hashiguchi, 1989](#_ENREF_17)) were also proposed based on the critical state soil mechanics ([Roscoe et al., 1963](#_ENREF_54)), in which NCL and CSL can be described as Eqs. (B1) and (B2) respectively:

 (B1)

 (B2)

where *η* is stress ratio, *λ* is compression index in the *e*-ln*p* plane, *e*N and *e*Г are the two reference void ratios at reference pressure *p*N (1 kPa) that determine the positions of NCL and CSL in the *e*-ln*p* plane.

Using the nonlinear interpolation in the modified Cam-Clay model ([Roscoe et al., 1963](#_ENREF_54)), the void ratio *e*, volumetric strain **v, and plastic volumetric strain  under a certain stress ratio *η* can be described as Eqs. (B3)–(B5):

 (B3)

 (B4)

 (B5)

where *κ* is swelling index in the *e*-ln*p* plane.

Eq. (B5) is essentially the basic form of the yield function, and the plastic volumetric strain increment is zero at the critical state:

 (B6)

which gives the  in terms of *λ* and *κ* as follows:

 (B7)

**Appendix C**

According to the NCL and the swelling line in the *e*-ln*p* plane, as shown in Fig. 1, the relationship between the void ratio *e* and the mean stress *p*m at current state can be obtained as:

 (C1)

where *λ* and *κ* are compression index and swelling index in the *e*-ln*p* plane respectively, *e*N is the reference void ratio at reference pressure *p*N (1 kPa) that determine the position of NCL in the *e*-ln*p* plane, *p*m and  represent the current positions of subloading surface and normal yield surface in the *p*-*q* plane.

Based on Eq. (C1), the initial normal consolidation stress  that determines the initial position of normal yield surface can be calculated with the initial mean stress *p*m0 and initial void ratio *e*0.

 (C2)