Full Length Article

**Mechanical and hydraulic properties of carbonate rock: The critical role of porosity**

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**Appendix A**

**Table A1.** Summary of empirical equations for uniaxial compressive strength (UCS in MPa) of carbonate rocks.

|  |  |  |  |
| --- | --- | --- | --- |
| Rock and condition | Regime | Equation | Source |
| Tight chalks | France, England and Belgium |  | Fay-gomord et al. (2016) |
| May provide the upper bound | Korobcheyev, Russia |  | Rzhevsky and Novick (1971) |
| Carbonated rocks | United Kingdom |  | Farquhar et al. (1994) |
| Low to moderate porosity (0.05 < *ϕ* <0.2) and high UCS  (30 MPa < *UCS* < 150 MPa) | Middle East |  | Chang (2004) |
| Low to moderate porosity  (0 < *ϕ* < 0.2) and high UCS  (10 MPa < *UCS* < 300 MPa) | Middle East |  | Chang (2004) |
| Carbonates for  0.117 < *ϕ* < 0.361 | France |  | Moh’d (2009) |
| Carbonates for *ϕ* < 0.3 | North Algeria |  | Hebib et al. (2017) |
| Lightened gypsum | Spain |  | Astorqui et al. (2017) |
| Asmari limestone | Iran |  | Maryam et al. (2018) |

: Predicted uniaxial compressive strength (MPa), *ϕ*: Porosity in fraction, *S*w: Water saturation, *ρ*: density of the compound (g/cm3), and *ρ*b: bulk density (g/cm3).

**Table A2.** Summary of empirical equations for Young’s modulus (*E* in GPa) of carbonate rocks.

|  |  |  |  |
| --- | --- | --- | --- |
| Rock and condition | Regime | Equation | Source |
| Carbonated rocks | United Kingdom |  | Farquhar et al. (1994) |
| Tight chalks | France, England and Belgium |  | Fay-gomord et al. (2016) |
| Carbonates for 0 < *ϕ* < 0.3 | Saudi Arabia |  | Ameen et al. (2009) |
| *a'* is -88440, -147900 and -40640;  *b* is 52120, 75180 and 24270; and  *c* is 41.98, 45.76 and 162.1 for Nekorot Limestone, Aminadav Dolomite and Bina Limestone | Israel |  | Palchik (2011) |
| Carbonates with  10 MPa < *UCS* < 300 MPa | NA |  | Chang (2004) |
| Dolomite with  60 MPa < *UCS* <100 MPa | NA |  | Chang (2004) |
| Sarvak and Asmari limestone | Iran |  | Najibi et al. (2015) |
| UCS estimated using plumb empirical correlation | Iran |  | Afsari et al. (2009) |
| Carbonate rocks | Iran |  | Asef and Farrokhrouz (2010) |
| Aminadav dolomite | Israel |  | Palchik and Hatzor (2000) |
| Bina limestone | Israel |  | Palchik and Hatzor (2000) |

Note: NA: Not available, *ϕ*: Porosity in fraction, *UCS*: Uniaxial compressive strength (MPa), *d*m: Mean grain size (mm), *ρ*: density, and *ρ*b: Bulk density (g/cm3).

***Physical Model – Derivation***

For a simple cubic packing of spheres, with increasing gradual superposition, the limit of this model is when spherical caps in *x*, *y* and *z* begin interfering. This happens when *a* = *r*sin45° = 0.707*r*, that is 0.707. At this limit, contact areas remain circular.

*A close-up of a measuring tape

Description automatically generated with low confidence*

Fig. A1. A spherical particle with radius *r*.

The volume of spherical cap is

|  |  |
| --- | --- |
|  | (A1) |

where or .

Replacing *h* with *r* and *a* and using , Eq. (A1) becomes

|  |  |
| --- | --- |
|  | (A2) |

Considering the deformation of the six spherical caps, the volume of the deformed solid sphere becomes

|  |  |
| --- | --- |
|  | (A3) |

The total volume including void spaces is

|  |  |
| --- | --- |
|  | (A4) |

The porosity *ϕ* is a ratio of volume of void to total volume, and volume of void is the difference between *V*t and *V*s. Using Eqs. (A3) and (A4), porosity can be expressed as

|  |  |
| --- | --- |
|  | (A5) |

Replacing *h* with *r* and *a* and using in Eq. (A5), porosity becomes

|  |  |
| --- | --- |
|  | (A6) |

For the maximum limit of , the lowest porosity using Eq. (A6) is *ϕ* = 0.035. The relative contact area *A*r is a ratio of projected circular contact area to the sectional area of maximum deformed solid sphere (i.e., a cube)

|  |  |
| --- | --- |
|  | (A7) |

Rearranging Eq. (A7), we have

|  |  |
| --- | --- |
|  | (A8) |

Substituting Eq. (A8) into Eq. (A6), a relationship between porosity *ϕ* and relative contact area *A*r is established

|  |  |
| --- | --- |
|  | (A9) |

Fig. A2 shows that the trend between porosity *ϕ* and relative contact area *A*rgiven by Eq. (17) or (A9)is very similar to the power function adopted for stiffness, strength, and brittle-to-ductile transition.



Fig. A2. A trend between a relative area *A*rgiven by Eq. (17) or (A9) and porosity *ϕ*.

**List of abbreviations and symbols**

|  |  |
| --- | --- |
| *UCS* | unconfined compressive strength |
| *UCS*0 | unconfined compressive strength at zero porosity |
| *a* | radius of contact area of particles |
| *A*r | relative contact area |
| *e* | void ratio |
| *e*\* | reference value for e |
| *E* | Young’s modulus |
| *E*0 | mean stiffness at zero porosity |
| *k* | permeability |
| *k*\* | reference value for *k* |
|  | mean effective principal stress |
| *P* | mechanical property below the percolation porosity |
| *P*0 | mechanical property at the percolation porosity |
|  | mean difference in effective principal stresses |
|  | deviatoric stress at failure |
| *r* | particle radius |
| *ϕ* | porosity |
| *φ*\* | limiting porosity |
| *φ*c | critical porosity |
| *φ*sc | porosity of a simple cubic packing of monosized particles |
|  | angle of internal shear strength |
| *ρ*b | dry bulk density |
| *ρ*m | mineral density |
|  | effective major principal stress |
|  | effective normal stress at failure |
|  | minor principal stress |
| *σ*bd | confining stress at the brittle-to-ductile transition |
|  | effective shear stress |
| *α* | fitted exponential coefficient |
| *β* | fitted exponential coefficient |
| *ψ* | fitted exponential coefficient |
| *μ* | mean value |